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Arrhythmia Detection Coefficient based on Wavelet Shrinkage

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Abstract

Electrocardiography (ECG) is the process of recording the electrical activity of the heart over a period of time using electrodes placed on the skin. The time intervals between its various peaks, may contain useful information about the nature of disease afflicting the heart. In the last years, different methods have been used by the researchers to detect arrhythmias from electrocardiogram (ECG) signal. In this paper we propose an arrhythmia detection coefficient based on the coefficient of variation and on the principle of wavelet shrinkage. This coefficient is tested in RR time series from databases of the PhysioBank.

Keywords: Arrhythmia Detection, Wavelet Shrinkage, ECG

I. Introduction

A wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. The nonparametric regression is the most important application of wavelet to the statistic, and it is based on the *principle of wavelet shrinkage*, which aims to reduce (and even remove) the noise present in a signal (see Donoho and Johnstone, 1994; Donoho and Johnstone, 1995; Donoho et al., 1995; Vidakovic, 1999 and Vargas and Veiga, 2017). The wavelet transform splits the data into lowpass (approximation) portions and highpass (detail) portions. *Wavelet shrinkage* reduces the magnitude of terms in the highpass portions. Finally, the wavelet transform is inverted to get the denoised version of the data.

Electrocardiography (ECG) is the process of recording the electrical activity of the heart over a period of time using electrodes placed on the skin. The time intervals between its various peaks, may contain useful information about the nature of disease afflicting the heart. The RR time series is the series of heartbeat interval, where R is a peak point respect to each heartbeat of the electrocardiography (ECG) wave, and RR is the interval between successive R. Different techniques have been used by the researchers in recent years to detect arrhythmias from electrocardiogram(ECG) signal (see Albuquerque et al., 2018; Elhaj et al., 2016; Alickovic and Subasi, 2016; Marwin, 2015; Taizhi et al., 2014 and Gallet et al., 2013).In this paper we proposed the CADWS, a Coefficient of Arrhythmias Detection based on the Wavelet Shrinkage and on the coefficient of variation. To test it we compared 16 healthy versus 16 unhealthy (with cardiac arrhythmia) RR time series from MIT-BIH database. It is proposed the CADWS as an additional diagnostic tool for cardiac arrhythmia.

The paper is organized as follows. Section II provides a background on wavelet analysis. The CADWS coefficient is proposed in Section III. In Section IV we present the analysis of the 16 healthy and 16 unhealthy (with cardiac arrhythmia) RR time series. Section V gives the conclusions.

II. Wavelet

In this section we give a brief overview on wavelets, presenting the discrete wavelet transforms (DWT), the wavelet shrinkage principal (see Donoho and Johnstone, 1994; Donoho et al., 1995 and Vidakovic, 1999).Two functions are very important in the wavelet analysis: the mother and father wavelets. These wavelets generate a family of functions that can reconstruct a signal.

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Definition 2.1 (Mother and Father Wavelets).A *mother wavelet* $\psi(\cdot)$ and a *father wavelet* (*or scale function*) $\phi(\cdot)$ are real functions $\psi, \phi: \mathbb{R} \to \mathbb{R}$ such that

$$\int_{\mathbb{R}} \psi(t)dt = 0, \int_{\mathbb{R}} \phi(t)dt = 1$$

and satisfy some integrability conditions, that is, $\psi, \phi \in L^2(\mathbb{R}) \cap L^1(\mathbb{R})$.

Given the wavelets $\psi(\cdot)$and $\phi(\cdot)$, we construct wavelet sequences through translations and dilatations of mother and father wavelets, respectively, given by

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}}\psi(2^{-j}t - k),$$
$$\phi_{j,k}(t) = 2^{-\frac{j}{2}}\phi(2^{-j}t - k).$$

The functions $\{\psi_{j,k}(\cdot), j, k \in \mathbb{Z}\}$ and $\{\phi_{j,k}(\cdot), j, k \in \mathbb{Z}\}$ form bases that are not necessarily orthogonal. The advantage of working with orthogonal bases is that they allow the perfect reconstruction of a signal from the coefficients of the transform. In general, the most used orthogonal wavelets are: Haar, Daublets, Symmlets and Coiflets.

Definition 2.2 (Discrete Wavelet Transform). Let $\boldsymbol{X} = (X_1, X_2, \cdots, X_n)'$ be an i.i.d. random sample, with$J = \lfloor \log_2(n) \rfloor$, where $\lfloor \cdot \rfloor$ indicates the integer part function. The *discrete wavelet transform* (DWT) of $\boldsymbol{X}$, with respect to the mother wavelet $\psi(\cdot)$, is defined as

$$d_{j,k} = \sum_{t=1}^{n} X_t \psi_{j,k}(t), \qquad (2.1)$$

for all $j = 1,2, \cdots, J$ and $k = 1,2, \cdots, \left\lfloor \frac{n}{2^j} \right\rfloor$. We can write the transform (3) in matrix form by

$$\boldsymbol{d}_j = \boldsymbol{W}_j \boldsymbol{X}, \qquad (2.2)$$

where $\boldsymbol{W}_j = \left(\psi_{j,k}(t)\right)_{k,t}$ is a $\left\lfloor \frac{n}{2^j} \right\rfloor \times n$ matrix. Assuming appropriate boundary conditions, the transform is orthogonal,and one can obtain the *inverse discrete wavelet transform* (IDWT) given by

$$\boldsymbol{X} = \boldsymbol{W}'\boldsymbol{d},$$

where$\boldsymbol{W}'$ denotes the transpose of $\boldsymbol{W}$.

To compute the DWT, one does not actually perform the matrix multiplication (2.2). Instead, one uses a fast "pyramid" algorithm with complexity $O(n)$ (see Meyer, 1993).

Wavelet shrinkage usually refers to reconstructions obtained from the shrunk wavelet coefficients. Let the simplest regression model

$$\boldsymbol{y_j} = \boldsymbol{f}(\boldsymbol{t_j}) + \boldsymbol{\epsilon_j}, \qquad \boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \cdots, \boldsymbol{n}, \qquad (2.3)$$

where the t_j's are equally spaced points and the ϵ_j's are independent Gaussian random variables with zero mean and variance σ_ϵ^2.Donoho and Johnstone (1994) and Donoho et al. (1995) have proposed a simple recipe based on thre sholding in the wavelet domain. Their wavelet estimation procedure has three steps: Take the discrete wavelet transform of the observations y_j for all $j \in \{1,2, \cdots, n\}$. Obtain, the coefficients without noise and apply the *inverse discrete wavelet transform* using the detail coefficients to recover the estimator of the function, $\hat{f}_j(\cdot)$, for all $j \in \{1,2, \cdots, n\}$.

III. CADWS coefficient

Here we present the coefficient of arrhythmias detection (CADWS) based on the coefficient of variation and on the principle of wavelet shrinkage (see Donoho and Johnstone, 1994; Donoho and Johnstone, 1995 and Donoho et al., 1995).

To obtain the CADWS coefficient to a given RR time series $\{X_t\}_{t=1}^{n}$, it is necessary the following steps.

The first step consists to apply the wavelet shrinkage procedure, that is, transform the observations X_i, $i \in \{1,2,\cdots,n\}$, into the symmlet wavelet ``s8" domain by applying a discrete wavelet transform (see definition 2.2), with level $J = \lfloor \log_2(g(n) - 3) \rfloor$, to obtain a sequence of wavelet coefficients $\boldsymbol{d}_4, \boldsymbol{d}_5, \cdots, \boldsymbol{d}_J$. Then shrink the wavelet coefficients towards zero, to obtain new detail coefficients $\tilde{d}_4 = \delta^H_{-\lambda}(d_4), \cdots, \tilde{d}_J = \delta^H_{-\lambda}(d_J)$, where $\lambda = \hat{\sigma}\sqrt{2\log(g(n) - 3)}$, $\hat{\sigma}$ is the estimated level of noise given by

$$\hat{\sigma} = \frac{median\{|d_{J-1,k}|: 0 \le k < 2^J\}}{0.6745}$$

and the δ^H_λ is the *hard* (H) *shrinkage* function defined by

$$\delta^H_\lambda = \begin{cases} 0, & if|x| \le \lambda \\ x, & if|x| > \lambda. \end{cases}$$

Then apply the inverse discrete wavelet transform, to get the *wavelet shrinkage estimator* $\hat{X}_i$ of X_i, for all $i \in \{1,2,\cdots,n\}$.

Finally we have the CADWS coefficient based on the coefficient of variation and on the wavelet shrinkage procedure,

$$\text{CADWS} = \frac{100}{\bar{X}}\left(\sum_{i=1}^{n} \frac{|\hat{X}_i - \tilde{X}|}{n}\right)$$

where $\hat{X}_i$ is the *wavelet shrinkage estimator* of X_i, $\tilde{X}$ is the median of $\{X_i\}_{i=1}^n$ and $\bar{X} = \sum_{i=1}^n \frac{X_i}{n}$.

IV. Application

The RR time series is the series of heartbeat interval, where R is a peak point respect to each heartbeat of the electrocardiography (ECG) wave, and RR is the interval between successive R. In this section, in view of the CADWS coefficient (see Section II), we analyze RR time series of 16 healthy and 16 unhealthy (with cardiac arrhythmia) selected from databases available from PhysioBank (https://www.physionet.org/cgi-bin/atm/ATM) in order to check the difference between healthy and unhealthy RR time series. In the Physion Bank for the healthy RR time series we use the database named as *MIT-BIH Normal Sinus Rhythm Database* ***(nsrdb)*** and for unhealthy RR time series we use database named as *MIT-BIH Arrhythmia Database* ***(mitdb)***. The results are presented in Table 4.1. From Table 4.1 we can see that for *healthy* RR time series we have always CADWS≥ 8. While for *unhealthy* RR times series we have always CADWS< 8.

Table 4.1: Results of the CADWS coefficient in RR Time Series of Healthy and Unhealthy Records.

Healthy Record in the nsrdb		Unhealthy Record in the mitdb	
Record's Number	CADWS	Record's Number	CADWS
16265	16,33	102	1,85
16272	9,43	103	1,86
16273	10,28	104	0,10
16420	8,92	105	3,00
16483	10,44	106	7,41
16539	10,41	107	7,41
16773	14,56	108	4,47
16786	10,96	109	4,47
16786	10,96	111	1,46
16795	14,01	112	1,88
17052	13,48	113	2,09
17453	9,51	114	7,31
18177	9,66	115	3,12
18184	8,15	116	2,25
19093	9,36	117	0,54
19090	8,79	118	5,03

V. Conclusions

We propose the CADWS coefficient, based on the coefficient of variation and on the wavelet shrinkage procedure. The CADWS coefficient is like an additional diagnostic tool that may provide an indication of cardiac arrhythmia. To test it we compared 16 healthy versus 16 unhealthy (with cardiac arrhythmia) RR time series randomly selected from MIT-BIH database. For all *healthy* RR time series we observe CADWS≥ 8. While for *unhealthy* RR times series we have always CADWS< 8.

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