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Piecewise Linear Economic-Mathematical Models with Regard to Unaccounted Factors Influence in 3-Dimensional Vector Space

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Abstract

For the last 15 years in periodic literature there has appeared a series of scientific publications that has laid the foundation of a new scientific direction on creation of piecewise-linear economic-mathematical models at uncertainty conditions in finite dimensional vector space. Representation of economic processes in finite-dimensional vector space, in particular in Euclidean space, at uncertainty conditions in the form of mathematical models in connected with complexity of complete account of such important issues as: spatial in homogeneity of occurring economic processes, incomplete macro, micro and social-political information; time changeability of multifactor economic indices, their duration and their change rate. The above-listed one in mathematical plan reduces the solution of the given problem to creation of very complicated economic-mathematical models of nonlinear type. In this connection, it was established in these works that all possible economic processes considered with regard to uncertainty factor in finite-dimensional vector space should be explicitly determined in spatial-time aspect. Owing only to the stated principle of spatial-time certainty of economic process at uncertainty conditions in finite dimensional vector space it is possible to reveal systematically the dynamics and structure of the occurring process. In addition, imposing a series of softened additional conditions on the occurring economic process, it is possible to classify it in finite-dimensional vector space and also to suggest a new science-based method of multivariate prediction of economic process and its control in finite-dimensional vector space at uncertainty conditions, in particular, with regard to unaccounted factors influence.

Keywords: Finite-dimensional vector space; Unaccounted factors; Unaccounted parameters influence function; Principle of certainty of economic process in fine-dimensional space; Multi alternative forecasting; Principle of spatial-time certainty of economic process at uncertainty conditions in fine-dimensional space; Piecewise-linear economic-mathematical models in view of the factor of uncertainty in finite-dimensional vector space; Piecewise-linear vector-function; 3-Dimensional Vector Space; 2-Component Piecewise-Linear Economic-Mathematical Model in 3-Dimensional Vector Space; Hyperbolic surface.

I. Introduction. Formulation of the problem

In publications [1-5,12] theory of construction of piecewise-linear economic mathematical models with regard to unaccounted factors influence in finite-dimensional vector space was developed. A method for predicting economic process and controlling it at uncertainty conditions, and a way for defining the economic process control function in m-dimensional vector space, were suggested.

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In addition to this we should note that no availability of precise definition of the notion “uncertainty” in economic processes, incomplete classification of display of this phenomenon, and also no availability of its precise and clear mathematical representation places the finding of the solution of problems of prediction of economic process and this control to the higher level by its complexity. Many-dimensionality and spatial in homogeneity of the occurring economic process, time changeability of multifactor economic indices and also their change velocity give additional complexity and uncertainty. Another complexity of the problem is connected with reliable construction of such a predicting vector equation in the consequent small volume $\Delta V_{n+1}(x_1, x_2, \ldots x_m)$ of finite-dimensional vector-space that could sufficiently reflect the state of economic process in the subsequent step. In other words, now by means of the given statistical points (vectors) describing certain economic process in the preceding volume $V = \sum_{N=1}^{N} \Delta V_N (x_1, x_2, \ldots x_m)$ of finite-dimensional vector space R_m one can construct a predicting vector equation $\vec{Z}_{n+1}(x_1, x_2, \ldots x_m)$ in the subsequent small volume $\Delta V_{n+1}(x_1, x_2, \ldots x_m)$ of finite-dimensional vector space. The goal of our investigation is to formulate the notion of uncertainty for one class of economical processes and also to find mathematical representation of the predicting function $\vec{Z}_{n+1}(x_1, x_2, \ldots x_m)$ for the given class of processes depending on so-called unaccounted factors functions. In connection with what has been said, below we suggest a method for constructing a predicting vector equation $\vec{Z}_{n+1}(x_1, x_2, \ldots x_m)$ in the subsequent small volume $\Delta V_{n+1}(x_1, x_2, \ldots x_m)$ of finite-dimensional vector space [1-7, 14].

II. Materials and methods:

In these publications, the postulate spatial-time certainty of economic process at uncertainty conditions in finite-dimensional vector space” was suggested, the notion of piecewise-linear homogeneity of the occurring economic process at uncertainty conditions was introduced, and also a so called. The unaccounted parameters influence function $\omega_n(\lambda_n^{k_n}, \alpha_{n-1,\mathrm{n}})$ influencing on the preceding volume $V = \sum_{N=1}^{N} \Delta V_n$ of economic process was suggested.

$$\vec{z}_n = \vec{z}_1 \left\{ 1 + A \left[1 + \omega_n(\lambda_n, \alpha_{n-1,n}) + \sum_{i=2}^{n-1 \geq 2} \omega_i (\lambda_i^{k_i}, \alpha_{i-1,i}) \right] \right\} \quad (1)$$

Here

$$\omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) = \lambda_i^{k_i} cos\alpha_{i-1,i} =$$

$$= \frac{\mu_i^{k_i}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{i-1} - \vec{z}_{i-1}^{k_{i-1}}\right| \left|\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}}\right|}{\vec{z}_1(\vec{z}_{i-1} - \vec{z}_{i-1}^{k_{i-1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\alpha_{i-1,i} \quad (2)$$

$$\mu_i = (\mu_{i-1} - \mu_{i-1}^{k_{i-1}}) \frac{(\vec{a}_i - \vec{z}_{i-2}^{k_{i-2}})(\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}})}{(\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}})^2}, \text{ for } \mu_{i-1} \geq \mu_{i-1}^{k_{i-1}} \quad (3)$$

$$\omega_n(\lambda_n, \alpha_{n-1,n}) = \lambda_n cos\alpha_{n-1,n} =$$

$$= \frac{\mu_n}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{n-1} - \vec{z}_{n-1}^{k_{n-1}}\right| \left|\vec{a}_{n+1} - \vec{z}_{n-1}^{k_{n-1}}\right|}{\vec{z}_1(\vec{z}_{n-1} - \vec{z}_{n-1}^{k_{n-1}})} \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\alpha_{n-1,n} \quad (3.1)$$

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \quad (4)$$

$$\mu_n=(\mu_{n-1}-\mu_{n-1}^{k_{n-1}})\frac{(\vec{a}_n-\vec{z}_{n-2}^{k_{n-2}})(\vec{a}_{n+1}-\vec{z}_{n-1}^{k_{n-1}})}{(\vec{a}_{n+1}-\vec{z}_{n-1}^{k_{n-1}})^2},\ \mu_{n-1}\geq\mu_{n-1}^{k_{n-1}} \tag{5}$$

On this basis, it was suggested the dependence of the n-th piecewise-linear function $\vec{z}_n$ on the first piecewise-linear function $\vec{z}_1$ and all spatial type unaccounted parameters influence function $\omega_n(\lambda_n^{k_n},\alpha_{n-1,n})$ influencing on the preceding interval of economic process, in the form Eqs. (1)–(5):

$$\vec{z}_n=\vec{z}_1\left\{1+A\left[1+\omega_n(\lambda_n,\alpha_{n-1,n})+\sum_{i=2}^{n-1\geq 2}\omega_i(\lambda_i^{k_i},\alpha_{i-1,i})\right]\right\} \tag{6}$$

Where

$$\omega_i(\lambda_i^{k_i},\alpha_{i-1,i})=\lambda_i^{k_i}cos\alpha_{i-1,i}=$$

$$=\frac{\mu_i^{k_i}}{\mu_1^{k_1}-\mu_1}\cdot\frac{|\vec{z}_{i-1}-\vec{z}_{i-1}^{k_{i-1}}||\vec{a}_{i+1}-\vec{z}_{i-1}^{k_{i-1}}|}{\vec{z}_1(\vec{z}_{i-1}-\vec{z}_{i-1}^{k_{i-1}})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{|\vec{a}_2-\vec{a}_1||\vec{z}_1^{k_1}-\vec{a}_1|}cos\alpha_{i-1,i} \tag{7}$$

are unaccounted parameters influence functions influencing on the preceding ΔV_1, ΔV_2,.. ΔV_i small volumes of economic process;

$$\mu_i=(\mu_{i-1}-\mu_{i-1}^{k_{i-1}})\frac{(\vec{a}_i-\vec{z}_{i-2}^{k_{i-2}})(\vec{a}_{i+1}-\vec{z}_{i-1}^{k_{i-1}})}{(\vec{a}_{i+1}-\vec{z}_{i-1}^{k_{i-1}})^2},\qquad \text{for } \mu_{i-1}\geq\mu_{i-1}^{k_{i-1}} \tag{8}$$

are arbitrary parameters referred to the i-th piecewise-linear straight line. And the parameters μ_i are connected with the parameter μ_{i-1} referred to the (i-1)-th piecewise-linear straight line, in the form Eq. (8);

$$A=(\mu_1^{k_1}-\mu_1)\frac{|\vec{a}_2-\vec{a}_1||\vec{z}_1^{k_1}-\vec{a}_1|}{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)} \tag{9}$$

is a constant quantity;

$$\omega_n(\lambda_n,\alpha_{n-1,n})=\lambda_n cos\alpha_{n-1,n}=$$

$$=\frac{\mu_n}{\mu_1^{k_1}-\mu_1}\cdot\frac{|\vec{z}_{n-1}-\vec{z}_{n-1}^{k_{n-1}}||\vec{a}_{n+1}-\vec{z}_{n-1}^{k_{n-1}}|}{\vec{z}_1(\vec{z}_{n-1}-\vec{z}_{n-1}^{k_{n-1}})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{|\vec{a}_2-\vec{a}_1||\vec{z}_1^{k_1}-\vec{a}_1|}cos\alpha_{n-1,n} \tag{10}$$

is the expression of the unaccounted parameters influence function that influences on subsequent small volume ΔV_N of finite-dimensional vector space. And the parameter μ_n referred to the n- piecewise-linear straight line is of the form:

$$\mu_n=(\mu_{n-1}-\mu_{n-1}^{k_{n-1}})\frac{(\vec{a}_n-\vec{z}_{n-2}^{k_{n-2}})(\vec{a}_{n+1}-\vec{z}_{n-1}^{k_{n-1}})}{(\vec{a}_{n+1}-\vec{z}_{n-1}^{k_{n-1}})^2},\qquad \mu_{n-1}\geq\mu_{n-1}^{k_{n-1}} \tag{11}$$

Here the parameter μ_n is connected with the parameter μ_{n-1} of the preceding (n-1)-th piecewise-linear vector equation of the straightline in the form Eq. (11). Thus, in finite-dimensional vector space, the system of statistical points (vectors) is represented in the vector form in the form of N piecewise-linear straight lines depending on the vector function of the first piecewise-linear straight-line $\vec{z}_1=\lambda_1\vec{a}_1+\mu_1\vec{a}_2$, and also on the unaccounted parameters influence function $\omega_n(\lambda_n,\alpha_{n-1,n})$ in all the investigated preceding volume of finite-dimensional vector space R_m.

After that, in publications [6-11,13-15] a solution was found of solve a problem on prediction of economic process and its control at uncertainty conditions in finite-dimensional vector space. It became clear, that the unaccounted parameters influence functions $\omega_n(\lambda_n, \alpha_{n\text{-}1,\mathrm{n}})$ are integral characteristics of influencing external factors occurring in environment that are not a priori situated in functional chain of sequence of the structured model but render very strong functional influence both on the function and on the results of prediction quantities Eq. (6). It is impossible to fix such a cause by statistical means. This means that the investigated this or other economic process in finite dimensional vector space directly or obliquely is connected with many dimensionality and spatial inhomogenlity of the occurring economic process, with time changeability of multifactor economic indices, vector and their change velocity. This in its turn is connected with the fact that the used statistical data of economic process in finite-dimensional vector space are of inhomogeneous in coordinates and time unstationary events character.

We assume the given unaccounted factors functions $\omega_n(\lambda_n, \alpha_{n\text{-}1,\mathrm{n}})$ hold on all the preceding interval of finite-dimensional vector space, the uncertainty character of this class of economic process. In such a statement, the problem on prediction of economic event on the subsequent small volume ΔV_{N+1} of finite-dimensional vector space will be directly connected in the first turn with the enumerated invisible external facts fixed on the earlier stages and their combinations, i.e., the functions $\omega_n(\lambda_n, \alpha_{n\text{-}1,\mathrm{n}})$ that earlier hold in the preceding small volumes $\Delta V_1, \Delta V_2, ...\Delta V_N$ of finite-dimensional vector space. Therefore, by studying the problem on prediction of economic process on subsequent small volume ΔV_{N+1} it is necessary to be ready to possible influence of such factors. In connection with such a statement of the problem, let's investigate behavior of economic process in subsequent small volume ΔV_{N+1} finite-dimensional vector space under the action of the desired unaccounted parameters function $\omega_n(\lambda_n, \alpha_{n\text{-}1,\mathrm{n}})$ that was earlier fixed by us in preceding small volumes ΔV_n of finite-dimensional vector space, i.e., $\omega_2(\lambda_2, \alpha_{1,2})$, $\omega_3(\lambda_3, \alpha_{2,3})$, …., $\omega_N(\lambda_N, \alpha_{\mathrm{N}\text{-}1,N})$. In connection with what has been said, the problem on prediction of economic process and its control in finite-dimensional vector space may be solved by means of the introduced unaccounted parameters influence function $\omega_n(\lambda_n, \alpha_{n\text{-}1,n})$ in the following way. Construct the (N+1)-the vector equation of piecewise-linear straight line $\vec{z}_{N+1} = \vec{z}^{\,k_N}_{N} + \mu_{N+1}(\vec{a}_{N+2} - \vec{z}^{\,k_N}_{N})$ depending on the vector equation of the first piecewise-linear straight line $\vec{z}_1$ and the desired unaccounted parameter influence function $\omega_\beta(\lambda_\beta, \alpha_{\beta\text{-}1,\beta})$ That we have seen in one of the preceding small volumes $\Delta V_1, \Delta V_2, ...\Delta V_N$ of finite-dimensional vector space. For that in Eqs. (6)–(11) we change the index n by $(N+1)$ and get:

$$\vec{z}_{N+1} = \vec{z}_1 \left\{ 1 + A\left[1 + \sum_{i=2}^{N} \omega_i(\lambda_i^{k_i}, \alpha_{i\text{-}1,i}) + \omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) \right] \right\} \tag{12}$$

Here

$$\omega_i(\lambda_i^{k_i}, \alpha_{i\text{-}1,i}) = \lambda_i^{k_i} cos\alpha_{i\text{-}1,i} =$$

$$= \frac{\mu_i^{k_i}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{i\text{-}1} - \vec{z}_{i\text{-}1}^{\,k_{i\text{-}1}}\right| \left|\vec{a}_{i+1} - \vec{z}_{i\text{-}1}^{\,k_{i\text{-}1}}\right|}{\vec{z}_1(\vec{z}_{i\text{-}1} - \vec{z}_{i\text{-}1}^{\,k_{i\text{-}1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|} cos\alpha_{i\text{-}1,i} \tag{13}$$

$$\mu_i = (\mu_{i\text{-}1} - \mu_{i\text{-}1}^{k_{i\text{-}1}}) \frac{(\vec{a}_i - \vec{z}_{i\text{-}2}^{\,k_{i\text{-}2}})(\vec{a}_{i+1} - \vec{z}_{i\text{-}1}^{\,k_{i\text{-}1}})}{(\vec{a}_{i+1} - \vec{z}_{i\text{-}1}^{\,k_{i\text{-}1}})^2}, \qquad \mu_{i\text{-}1} \geq \mu_{i\text{-}1}^{k_{i\text{-}1}} \tag{14}$$

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)} \tag{15}$$

$$\omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) = \lambda_{N+1} cos\alpha_{N,N+1} =$$

$$=\frac{\mu_{N+1}}{\mu_1^{k_1}-\mu_1}\cdot\frac{\left|\vec{z}_N-\vec{z}_N^{k_N}\right|\left|\vec{a}_{N+2}-\vec{z}_N^{k_N}\right|}{\vec{z}_1(\vec{z}_N-\vec{z}_N^{k_N})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{\left|\vec{a}_2-\vec{a}_1\right|\left|\vec{z}_1^{k_1}-\vec{a}_1\right|}cos\alpha_{N,N+1} \quad (16)$$

$$\mu_{N+1}=(\mu_N-\mu_N^{k_N})\frac{(\vec{a}_{N+1}-\vec{z}_{N-1}^{k_{N-1}})(\vec{a}_{N+2}-\vec{z}_N^{k_N})}{(\vec{a}_{N+2}-\vec{z}_N^{k_N})^2},\ \mu_{N}\geq\mu_N^{k_N} \quad (17)$$

For the behavior of economic process on the subsequent small volume ΔV_{N+1} of finite-dimensional vector space to be as in one of the desired preceding ones in small volume ΔV_β it is necessary that the vector equations of piecewise-linear straight lines $\vec{z}_{N+1}$ and $\vec{z}_\beta$ to be situated in one of the planes of these vectors and to be parallel to one another, i.e.

$$\vec{z}_{N+1}=C\vec{z}_\beta \quad (18)$$

In connection with what has been said, to ΔV_{N+1} finite-dimensional space there should be chosen such a vector-point $\vec{a}_{N+2}$ that the piecewise-linear straight lines $\vec{z}_{N+1}=(\vec{a}_{N+2}-\vec{z}_N^{k_N})$ and $\vec{z}_\beta=(\vec{a}_{\beta+1}-\vec{z}_{\beta-1}^{k_{\beta-1}})$ could be situated in the same plane of these vectors and at the same time be parallel to each other (Fig. 1).

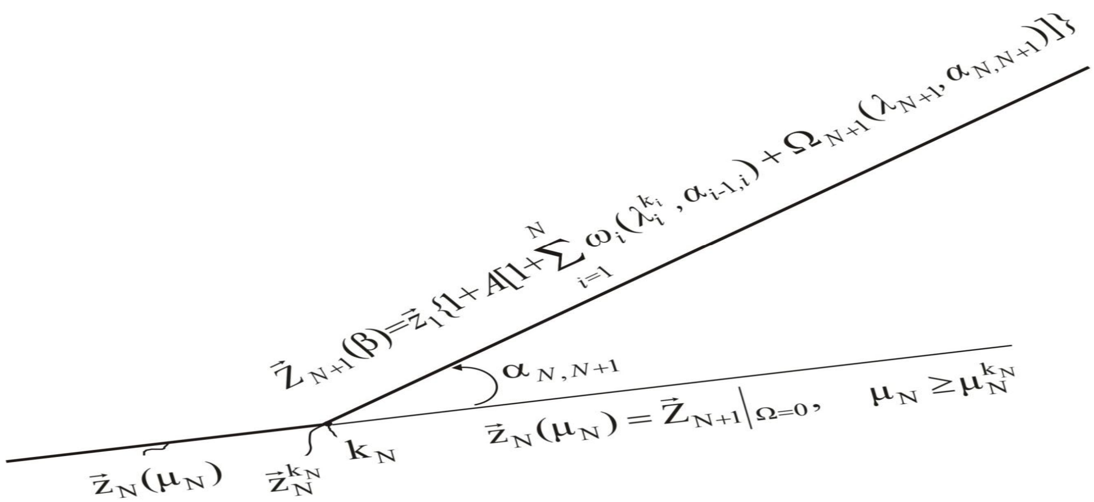


Fig. 1. The scheme of construction of prediction function of economic process $\vec{Z}_{N+1}(\beta)$ at uncertainty conditions in finite-dimensional vector space R_m. Prediction function $\vec{Z}_{N+1}(\beta)$ will lie in the same plane with one of the desired preceding β -the piecewise-linear straight line and will be parallel to it.

In other words, they should satisfy the following parallelism condition:

$$(\vec{a}_{N+2}-\vec{z}_N^{k_N})=C(\vec{a}_{\beta+1}-\vec{z}_{\beta-1}^{k_{\beta-1}}) \quad (19)$$

Here

$$\vec{a}_{N+2}=\sum_{m=1}^{M}a_{N+2,m}\vec{i}_m,\ \vec{a}_{\beta+1}=\sum_{m=1}^{M}a_{\beta+1,m}\vec{i}_m,$$

$$\vec{z}_N^{k_N}=\sum_{m=1}^{M}z_{N,m}^{k_N}\vec{i}_m,\ \vec{z}_{\beta-1}^{k_{\beta-1}}=\sum_{m=1}^{M}z_{\beta-1,m}^{k_{\beta-1}}\vec{i}_m$$

Excluding in Eq. (19) the parameter C, we get:

$$\frac{a_{N+2,1}-z_{N,1}^{k_N}}{a_{\beta+1,1}-z_{\beta-1,1}^{k_{\beta-1}}}=\frac{a_{N+2,2}-z_{N,2}^{k_N}}{a_{\beta+1,2}-z_{\beta-1,2}^{k_{\beta-1}}}=.....=\frac{a_{N+2,M}-z_{N,M}^{k_N}}{a_{\beta+1,M}-z_{\beta-1,M}^{k_{\beta-1}}} \quad (20)$$

It is easy to define from system Eq. (20) the coefficients of the vector $\vec{a}_{N+2}$:

$$a_{N+2,2}=z_{N,2}^{k_N}+\frac{a_{\beta+1,2}-z_{\beta-1,2}^{k_{\beta-1}}}{a_{\beta+1,1}-z_{\beta-1,1}^{k_{\beta-1}}}(a_{N+2,1}-z_{N,1}^{k_N})$$

$$a_{N+2,3}=z_{N,3}^{k_N}+\frac{a_{\beta+1,3}-z_{\beta-1,3}^{k_{\beta-1}}}{a_{\beta+1,1}-z_{\beta-1,1}}(a_{N+2,1}-z_{N,1}^{k_N})$$

$$a_{N+2,M}=z_{N,M}^{k_N}+\frac{a_{\beta+1,M}-z_{\beta-1,M}^{k_{\beta-1}}}{a_{\beta+1,M-1}-z_{\beta-1,M-1}^{k_{\beta-1}}}(a_{N+2,M-1}-z_{N,M-1}^{k_N}) \quad (21)$$

In this case, the vector $\vec{a}_{N+2}$ will have the following final form:

$$\vec{a}_{N+2}=a_{N+2,1}\vec{i}_1+a_{N+2,2}\vec{i}_2+a_{N+2,3}\vec{i}_3+......+a_{N+2,M}\vec{i}_M \quad (22)$$

As the coordinates of the point (of the vector) $\vec{a}_{N+2}$ now are determined by means of the piecewise-linear vector $\vec{z}_\beta=\vec{a}_{\beta+1}-\vec{z}_{\beta-1}^{k_{\beta-1}}$ taken from one of the preceding stage of economic process, it is appropriate to denote them in the form $\vec{a}_{N+2}(\beta)$ [8-13]. This will show that the coordinates of the point $\vec{a}_{N+2}$ (3) were determined by means of piecewise-linear straight line $\vec{z}_\beta$. In this case it is appropriate to represent Eq. (22) in the following compact form:

$$\vec{a}_{N+2}(\beta)=\sum_{m=1}^{M}a_{N+2,m}(\beta)\vec{i}_m \quad (23)$$

Now, in the system of Eqs. (12)–(17), instead of the vector $\vec{a}_{N+1}$ we substitute the value of the vector $\vec{a}_{N+2}(\beta)$, and also instead of $\omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})$ introduce the denotation of the so-called predicting influence function with regard to unaccounted parameters $\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})$. In this case the prediction function of the economic process $\vec{Z}_{N+1}(\beta)$ with regard to influence of prediction function of unaccounted parameters $\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})$ will take the following form:

$$\vec{Z}_{N+1}(\beta)=\vec{z}_1\left\{1+A\left[1+\sum_{i=2}^{N}\omega_i(\lambda_i^{k_i},\alpha_{i-1,i})+\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})\right]\right\} \quad (24)$$

Here

$$\omega_i(\lambda_i^{k_i},\alpha_{i-1,i})=\lambda_i^{k_i}cos\alpha_{i-1,i}=$$

$$=\frac{\mu_i^{k_i}}{\mu_1^{k_1}-\mu_1}\cdot\frac{\left|\vec{z}_{i-1}-\vec{z}_{i-1}^{k_{i-1}}\right|\left|\vec{a}_{i+1}-\vec{z}_{i-1}^{k_{i-1}}\right|}{\vec{z}_1(\vec{z}_{i-1}-\vec{z}_{i-1}^{k_{i-1}})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{\left|\vec{a}_2-\vec{a}_1\right|\left|\vec{z}_1^{k_1}-\vec{a}_1\right|}cos\alpha_{i-1,i} \quad (25)$$

$$\mu_i=(\mu_{i-1}-\mu_{i-1}^{k_{i-1}})\frac{(\vec{a}_i-\vec{z}_{i-2}^{k_{i-2}})(\vec{a}_{i+1}-\vec{z}_{i-1}^{k_{i-1}})}{(\vec{a}_{i+1}-\vec{z}_{i-1}^{k_{i-1}})^2},\quad \mu_{i-1}\geq\mu_{i-1}^{k_{i-1}} \quad (26)$$

$$A=(\mu_1^{k_1}-\mu_1)\frac{\left|\vec{a}_2-\vec{a}_1\right|\left|\vec{z}_1^{k_1}-\vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)} \quad (27)$$

And the prediction function of influence of unaccounted parameters $\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})$ will take the form:

$$\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})=\lambda_{N+1}cos\alpha_{N,N+1} \quad (28)$$

$$\lambda_{N+1}=\frac{\mu_{N+1}}{\mu_1^{k_1}-\mu_1}\cdot\frac{|\vec{z}_N-\vec{z}_N^{k_N}||\vec{a}_{N+2}(\beta)-\vec{z}_N^{k_N}|}{\vec{z}_1(\vec{z}_N-\vec{z}_N^{k_N})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{|\vec{a}_2-\vec{a}_1||\vec{z}_1^{k_1}-\vec{a}_1|} \quad (29)$$

$$\mu_{N+1}=(\mu_N-\mu_N^{k_N})\frac{(\vec{a}_{N+1}-\vec{z}_{N-1}^{k_{N-1}})(\vec{a}_{N+2}(\beta)-\vec{z}_N^{k_N})}{(\vec{a}_{N+2}(\beta)-\vec{z}_N^{k_N})^2},\ \mu_{N}\geq\mu_N^{k_N} \quad (30)$$

Here the vector $\vec{a}_{N+2}(\beta)$ is determined by Eq. (23).

Note the following points. It is seen from Eq. (11) that for $\mu_N=\mu_N^{k_N}$ the value of the parameter $\mu_{N+1}=0$. By this fact from Eq. (28) it will follow that the value of the predicting function of influence of unaccounted parameters $\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})$ will equal:

$$\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})=0 \text{ for } \mu_{N+1}=0$$
$$\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})\neq 0 \text{ for } \mu_{N+1}>0 \quad (31)$$

This will mean that the initial point from which the (N+1)-th vector equation of the prediction function of economic process $\vec{Z}_{N+1}(\beta)$ will enanimate, will coincide with the final point of the n-th vector equation of piecewise-linear straight line $\vec{z}_N$ and equal:

$$Z_{N+1}=\vec{z}_1\left\{1+A\left[1+\sum_{i=2}^{N}\omega_i(\lambda_i^{k_i},\alpha_{i-1,i})\right]\right\},\quad \text{for } \mu_{N+1}=0 \quad (32)$$

For any other values of the parameter $\mu_{N+1}\neq 0$ the points of the $(N+1)$-th vector equation will be determined by Eq. (24). It is seen from Eq. (28) that $\lambda_{N+1}=0$ and $\Omega_{N+1}(\lambda_{N+1}=0;\alpha_{N,N+1})=0$ will follow $cos\alpha_{N,N+1}=0$ and $\mu_{N+1}\neq 0$. This will correspond to the case when the influence of external unaccounted factors on subsequent small volume ΔV_{N+1} are as in the preceding small volume ΔV_N of finite-dimensional vector space. In this case it suffices to continue the preceding vector equation $\vec{z}_N$ to the desired point $\mu_{N+1}=\mu_{N+1}^*>\mu_N^{k_N}$ of subsequent small volume of finite-dimensional vector space.

The value of the vector function $\vec{Z}_{N+1}(\mu_{N+1}^*)=\vec{z}_N(\mu_{N+1}^*;\lambda_N,\alpha_{N-1,N})$ at the point $\mu_{N+1}=\mu_{N+1}^*$ will be one of the desired prediction values of economic process in subsequent small volume ΔV_{N+1}. In this case, the value of the controlled parameter of unaccounted factors will be equal to zero, i.e.,

$$\Omega_{N+1}(\mu_{N+1}\neq 0;\lambda_{N+1}\neq 0;\cos\alpha_{N,N+1}=0;\alpha_{N,N+1}=0)=0$$

For any other value of the parameter μ_{N+1}, taken on the interval $0\leq\ \mu_{N+1}\leq\ \mu_{N+1}^*$ and $\cos\alpha_{N,N+1}\neq 0$, the corresponding prediction function of unaccounted parameters will differ from zero, i.e., $\Omega_{N+1}(\lambda_{N+1},\alpha_{N,N+1})\neq 0$. Thus, choosing by desire the numerical values of unaccounted parameters function $\omega_\beta(\mu_{N+1};\lambda_\beta,\alpha_{\beta-1,\beta})=\Omega_{N+1}(\lambda_{N+1}^*,\alpha_{N,N+1})$ corresponding to preceding small volumes $\Delta V_1,\Delta V_2,....\Delta V_N$ and influencing by them beginning with the point $\mu_{N+1}=0$ to the desired point μ_{N+1}^*, we get numerical values of predicting economic event $\vec{Z}_{N+1}(\mu_{N+1}^*;\lambda_{N+1}^*,\alpha_{N,N+1})$ on subsequent step of the small volume ΔV_{N+1} (Fig. 2).

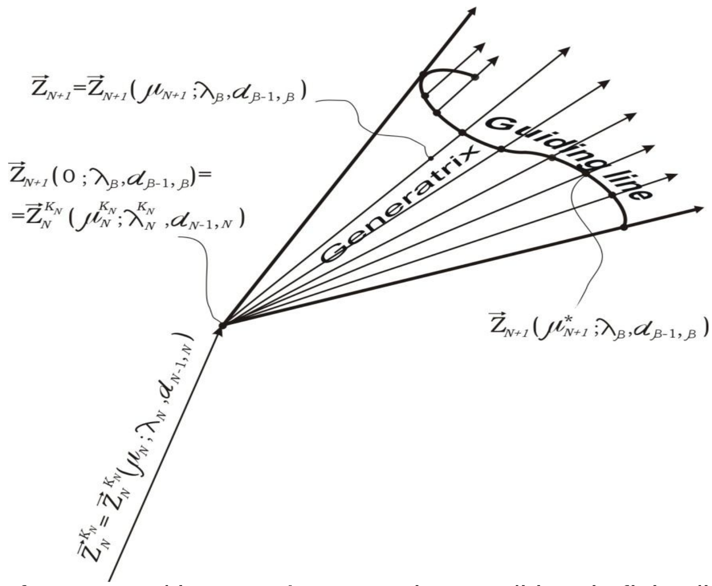


Fig. 2. The graph of prediction of process and its control at uncertainty conditions in finite-dimensional vector space. It is represented in the form of hypersonic surface whose points, of directrix will form the line of economic process prediction. Taking into account the fact that by desire we can choose the predicting influence function of unaccounted parameters $\Omega^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})$, then this function will represent a predicting control function of unaccounted factors, and its appropriate function $\vec{Z}^*_{N+1}(\mu^*_{N,N+1};\lambda^*_{N+1},\alpha_{N,N+1})$ will be a control aim function of economic event in finite-dimensional vector space. Speaking about unaccounted parameters prediction function $\Omega_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$
we should understand their preliminarily calculated values in previous small volumes $\Delta V_1,\Delta V_2,....\Delta V_N$ of finite-dimensional vector space. Therefore, in Eq. (24) we used calculated ready values of the function $\Omega_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$.Thus, influencing by the unaccounted parameters influence functions of the form $\Omega_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$ or by their combinations from the end of the vector equation of piecewise-linear straight line $\vec{z}^{\,k_N}_N(\mu^{k_N}_N;\ \lambda^{k_N}_N,\alpha_{N\text{-}1,N})$ situated on the boundary of the small volume $\vec{Z}_{N+1}(\beta)=\vec{Z}_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$ there will originate the vectors ΔV_N and ΔV_{N+1}, lying on the subsequent small volume ΔV_{N+1}.These vectors will represent the generators of hyperbolic surface of finite-dimensional vector space. The values of this series vector-functions for small values of the parameter $\mu_{N+1}=\overset{*}{\mu}_{N+1}$, i.e., $\vec{Z}_{N+1}(\mu^*_{N+1};\lambda_{N+1},\alpha_{N,N+1})$ will represent the points directrix of hyperconic surface of finite-dimensional vector space (Fig. 2). The series of the values of the points of directrix hyperconic surface will create a domain of change of predictable values of the function of $\vec{Z}^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})$ in the further step in the small volume ΔV_{N+1}.This predictable function will have minimum and maximum of its values $[\vec{Z}^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})]_{\min}$ and $[\vec{Z}^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})]_{max}$. Thus, the found domain of change of predictable function of economic process in the form $\vec{Z}_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})$, or in other words, the points of directrix of hyperbolic surface will represent the domain of economic process control in finite-dimensional vector-space.

III. 2-Component Piecewise-Linear Economic-Mathematical Model and Method of Multivariate Prediction of Economic Process With Regard to Unaccounted Factors Influence in 3-Dimensional Vector Space

In this article we give a number of practically important piecewise-linear economic-mathematical models with regard to unaccounted parameters influence factor in their-dimensional vector space. And by means of two- and three-component piecewise-linear models suggest an appropriate method of multivariant prediction of economic process in subsequent stages and its control then at uncertainty conditions in 3-dimensional vector space [6-11,13-15].

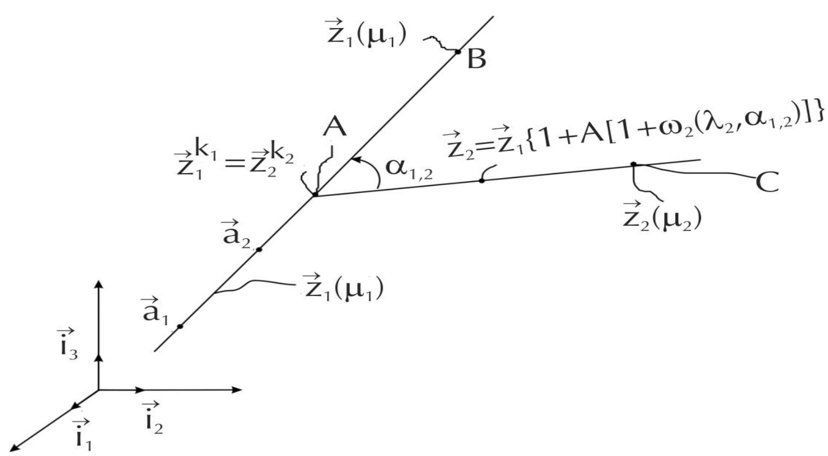
Given a statistical table describing some economic process in the form of the points (vector) set $\{\vec{a}_n\}$ of 3-dimensional vector space R_3. Here the numbers a_{ni} are the coordinates of the vector $\vec{a}_n$ (a_{n1}, a_{n2}, a_{n3}, …… a_{ni}). With the help of the points (vectors) $\vec{a}_n$ represent the set of statistical points in the vector form in the form of 2-component piecewise-linear function [1–6]:

$$\vec{z}_1 = \vec{a}_1 + \mu_1(\vec{a}_2 - \vec{a}_1) \qquad (33)$$

$$\vec{z}_2 = \vec{a}_2 + \mu_2(\vec{a}_3 - \vec{a}_2) \qquad (34)$$

Here $\vec{z}_1 = \vec{z}_1(z_{11}, z_{12}, z_{13})$ and $\vec{z}_2 = \vec{z}_2(z_{21}, z_{22}, z_{23})$ are the equations of the first and second piecewise-linear straight lines on 3-dimensional vector space; the vectors $\vec{a}_1(a_{11}, a_{12}, a_{13})$, $\vec{a}_2 = \vec{a}_2(a_{21}, a_{22}, a_{23})$ and $\vec{a}_3 = \vec{a}_3(a_{31}, a_{32}, a_{33})$ are the given points (vectors) in 3-dimensional space; $\mu_1 \geq 0$ and $\mu_2 \geq 0$ are arbitrary parameters of the first and second piecewise-linear straight lines. And it holds the equality $\lambda_1 + \mu_1 = 1$ and $\lambda_2 + \mu_2 = 1$; $\alpha_{1,2}$ is the angle between the piecewise-linear straight lines; k_1 is the intersection point between the first and second straight lines (Fig. 3). Note that in the general case, the intersection point of these straight lines may also not coincide with the point $\vec{a}_2$. Therefore, according to the conjugation condition $\vec{z}_1^{k_1} = \vec{z}_2^{k_1}$, we denote the intersection point between the first and second piecewise-linear straight lines in 3-dimensional vector space by $\vec{z}_1^{k_1}$ (Fig. 3). Allowing for this fact, we write the equation of the second piecewise-linear straight line in the form Eq. (35): $\vec{z}_2 = \vec{z}_1^{k_1} + \mu_2(\vec{a}_3 - \vec{z}_1^{k_1})$ (35) where the value $\vec{z}_1^{k_1}$ is the value of the point (vector) of the first piecewise-linear straight line at the k_1-the intersection point and equals:

Fig. 3. Construction of 2-component piecewise-linear economic-mathematical model



In 3-dimensional vector space R_3.

$$\vec{z}_1^{k_1} = \vec{a}_1 + \mu_1^{k_1}(\vec{a}_2 - \vec{a}_1) \qquad (36)$$

In particular case, $\vec{z}_1^{k_1} = \vec{a}_2$ for $\mu_1^{k_1} = 1$. In this case, the intersection point $\vec{z}_1^{k_1}$ coincides with the point $\vec{a}_2$. Now, according to Eqs. (1)–(11) of, the vector equation for the points of the second piecewise-linear straight line depending on the vector function of the first piecewise-linear straight line $\vec{z}_1$ and introduced unaccounted parameters influence spatial function $\omega_2(\lambda_2, \alpha_{1,2})$ in 3-dimensional vector space is written in the form (Fig. 3) [7–9]:

$$\vec{z}_2 = \vec{z}_1\{1+A\ [1+\omega_2(\lambda_2, \alpha_{1,2})]\} \quad (37)$$

Here

$$A = (\mu_1^{k_1} - \mu_1)\frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \quad (38)$$

$$\lambda_2 = \frac{\mu_2}{\mu_1^{k_1} - \mu_1} \cdot \frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|} \quad (39)$$

$$\omega_2(\lambda_2, \alpha_{1,2}) = \lambda_2 cos\alpha_{1,2} \quad (40)$$

$$\mu_2 = (\mu_1 - \mu_1^{k_1})\frac{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)}{(\vec{a}_3 - \vec{z}_1^{k_1})^2}, \text{ for } \mu_1 \geq \mu_1^{k_1} \quad (41)$$

Eq. (41) is the mathematical relation between arbitrary parameters μ_2 and μ_1. For the second piecewise-linear straight line, representing a straight line restricted with one end, condition Eq. (41) will hold for all $\mu_1 \geq \mu_1^{k_1}$. Furthermore, for the second intersection point k_2, i.e., for $\mu_2 = \mu_2^{k_2}$, the appropriate value of the parameter μ_1 will be determined as follows:

$$\mu_1^{k_2} = \mu_1^{k_1} + \frac{(\vec{a}_3 - \vec{z}_1^{k_1})^2}{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)}\mu_2^{k_2} \quad (42)$$

The value of $cos\alpha_{1,2}$ between the first and second piecewise-linear straight lines is determined by means of the scalar product of 2 vectors of the form (Fig. 3):

$$cos\alpha_{1,2} = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{(\vec{z}_1 - \vec{z}_1^{k_1})(\vec{z}_2 - \vec{z}_1^{k_1})}{|\vec{z}_1 - \vec{z}_1^{k_1}| \cdot |\vec{z}_2 - \vec{z}_1^{k_1}|} \quad (43)$$

By calculating the values of $cos\alpha_{1,2}$ we can use any values of arbitrary parameters μ_1 and μ_2.

Thus, in 3-dimensional vector space, determining the points (vectors):

$$\vec{a}_1 = \sum_{m=1}^{3} a_{1m}\vec{i}_m, \quad \vec{a}_2 = \sum_{m=1}^{3} a_{2m}\vec{i}_m, \quad \vec{a}_3 = \sum_{m=1}^{3} a_{3m}\vec{i}_m$$

$$\vec{z}_1^{k_1} = \sum_{m=1}^{3} z_{1m}^{k_1}\vec{i}_m = \sum_{m=1}^{3}[a_{1m} + \mu_1^{k_1}(a_{2m} - a_{1m})]\vec{i}_m \quad (44)$$

Eq. (37) will represent an equation for the second vector straight line $\vec{z}_2 = \vec{z}_2(\mu_1, \omega_2)$ depending on the unaccounted parameter influence function $\omega_2(\lambda_2, \alpha_{1,2})$ and arbitrary parameter $\mu_1 \geq \mu_1^{k_1}$. Represent the vector equation for the second piecewise-linear straight line Eq. (37) in the coordinate form. For that take into account that in 3-dimensional space the vectors of the first and second piecewise-linear straight lines in the coordinate form are represented in the form:

$$\vec{z}_1 = \sum_{m=1}^{3} z_{1m}\vec{i}_m \text{ and } \vec{z}_2 = \sum_{m=1}^{3} z_{2m}\vec{i}_m \quad (45)$$

In this case, the coordinates of the vector $\vec{z}_2$ Eq. (37), i.e., z_{2m} will be expressed by the coordinates of the first piecewise-linear vector z_{1m}, spatial vector λ_2 and the unaccounted parameter influence function $\omega_2(\lambda_2, \alpha_{1,2})$, in the form:

$$z_{2m} = \{1 + A[1 + \omega_2(\lambda_2, \alpha_{1,2})]\} z_{1m}, \text{ for } m = 1,2,3 \tag{46}$$

Here the coordinate notation of the coefficients A, λ_2 and $\omega_2(\lambda_2, \alpha_{1,2})$, by Eqs. (38)–(41), will be of the form:

$$A = (\mu_1^{k_1} - \mu_1) \frac{\sum_{i=1}^{3}(a_{2i} - a_{1i})^2}{\sum_{i=1}^{3}(a_{2i} - a_{1i})[a_{1i} + \mu_1(a_{2i} - a_{1i})]} \tag{47}$$

$$\lambda_2 = \frac{\mu_2}{\mu_1^{k_1} - \mu_1} \cdot \frac{\sqrt{\sum_{i=1}^{3}\{a_{3i} - [a_{1i} + \mu_1^{k}(a_{2i} - a_{1i})]\}^2}}{\sqrt{\sum_{i=1}^{3}(a_{2i} - a_{1i})^2}} \tag{48}$$

$$\omega_2(\lambda_2, \alpha_{1,2}) = \lambda_2 cos\alpha_{1,2} \tag{49}$$

$$\mu_2 = (\mu_1 - \mu_1^{k_1}) \frac{\sum_{i=1}^{3}(a_{2i} - a_{1i})[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]}{\sum_{i=1}^{3}[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]^2}, \quad \text{for } \mu_1 \geq \mu_1^{k_1} \tag{50}$$

$$\mu_2^{k_2} = \frac{\sum_{i=1}^{3}(a_{2i} - a_{1i})[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]}{\sum_{i=1}^{3}[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]^2} (\mu_1^{k_2} - \mu_1^{k_1}) \tag{51}$$

Now, for the case economic process represented in the form of 2-component piecewise-linear economic-mathematical model, investigate the prediction and control of such a process on the subsequent $\Delta V_3(x_1, x_2, x_3)$ small volume of 3-dimensional vector space with regard to unaccounted parameter influence function $\omega_2(\lambda_2, \alpha_{1,2})$. And the value of the unaccounted parameter $\omega_2(\lambda_2, \alpha_{1,2})$ function is assumed to be known [6-11,13-15]. A method for constructing a predicting vector function of economic process $\vec{Z}_{N+1}(\beta)$ with regard to the introduced unaccounted parameters influence predicting function $\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1})$ in m-dimensional vector space, represented by Eqs. (24)–(30) was developed above. Apply this method to the case of the given 2-component piecewise-linear economic model 3-dimensional vector space. It will be of the form:

$$\vec{Z}_3(1) = \vec{z}_1\{1 + A[1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}) + \Omega_3(\lambda_3, \alpha_{2,3})]\} \tag{52}$$

Where

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = \lambda_2^{k_2} \cdot cos\alpha_{1,2} =$$

$$= \frac{\mu_2^{k_2}}{\mu_1^{K_1} - \mu_1} \cdot \frac{\left|\vec{z}_1 - \vec{z}_1^{k_1}\right| \cdot \left|\vec{a}_3 - \vec{z}_1^{k_1}\right|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \cdot \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\alpha_{1,2} \tag{53}$$

$$\mu_2 = (\mu_1 - \mu_1^{k_1}) \cdot \frac{(\vec{a}_2 - \vec{a}_1)(\vec{a}_3 - \vec{z}_1^{k_1})}{(\vec{a}_3 - \vec{z}_1^{k_1})^2}, \quad \mu_1 \ge \mu_1^{k_1} \tag{54}$$

$$A = (\mu_1^{k_1} - \mu_1) \cdot \frac{\left|\vec{a}_2 - \vec{a}_1\right| \cdot \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{55}$$

$$\Omega_3(\lambda_3, \alpha_{2,3}) = \lambda_3 \cdot cos\alpha_{2,3} \tag{56}$$

$$\lambda_3 = \frac{\mu_3}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_2 - \vec{z}_2^{k_2}\right| \cdot \left|\vec{a}_4(1) - \vec{z}_2^{k_2}\right|}{\vec{z}_1(\vec{z}_2 - \vec{z}_2^{k_{21}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \cdot \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} \tag{57}$$

$$\mu_3 = (\mu_2 - \mu_2^{k_2}) \cdot \frac{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_4(1) - \vec{z}_2^{k_2})}{(\vec{a}_4(1) - \vec{z}_2^{k_2})^2}, \quad \mu_2 \ge \mu_2^{k_2},\ \mu_3 \ge 0 \tag{58}$$

Here, according to Eq. (40), the vector $\vec{a}_4(\beta)$ is of the form:

$$\vec{a}_4(1) = a_{41}(1)\vec{i}_1 + a_{42}(1)\vec{i}_2 + a_{43}(1)\vec{i}_3 = \sum_{m=1}^{3} a_{4m}(1) \cdot \vec{i}_m \tag{59}$$

And the coordinates of a_{42} and a_{43} are expressed by the arbitrarily given coordinate $a_{41} > z_{21}^{k_2}$ in the form:

$$C = \frac{a_{41}(1) - z_{21}^{k_2}}{a_{21} - a_{11}} = \frac{a_{42}(1) - z_{22}^{k_2}}{a_{22} - a_{12}} = \frac{a_{43}(1) - z_{23}^{k_2}}{a_{23} - a_{13}} \tag{60}$$

Hence:

$$a_{42}(1) = z_{22}^{k_2} + \frac{a_{22} - a_{12}}{a_{21} - a_{11}}(a_{41}(1) - z_{21}^{k_2})$$

$$a_{43}(1) = z_{23}^{k_3} + \frac{a_{23} - a_{13}}{a_{21} - a_{11}}(a_{41}(1) - z_{21}^{k_2}) \tag{61}$$

Here the coefficients a_{2m}, a_{1m} and $z_{2m}^{k_2}$ are the coordinates of the vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{z}_2^{k_2}$ in 3-dimensional vector space and equal:

$$\vec{a}_1 = \sum_{m=1}^{3} a_{1m} \cdot \vec{i}_m\ ,\ \vec{a}_2 = \sum_{m=1}^{3} a_{2m} \cdot \vec{i}_m\ ,\ \vec{z}_2^{k_2} = \sum_{m=1}^{3} \vec{z}_{2m}^{k_2} \vec{i}_m \tag{62}$$

Note that in the vectors $\vec{Z}_3(1)$ and $\vec{a}_4(1)$ the index (1) in the brackets means that the vector $\vec{Z}_3(1)$ is parallel to the first piecewise-linear vector function $\vec{z}_1$. This means that the economic process beginning with the point $\vec{z}_2^{k_2}$ will hold by the scenario of the first piecewise-linear equation (Fig. 4).

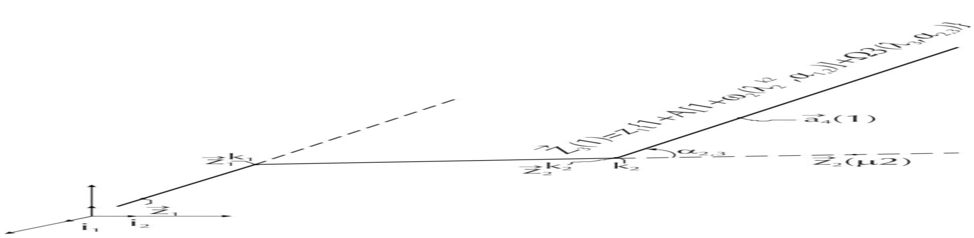


Fig. 4. Construction of the predicting vector function $\vec{Z}_3(\beta)$ with regard to unaccounted parameter influence predicting function $\Omega_3(\lambda_3, \alpha_{2,3})$ on the base of 2-component economic-mathematical model in 3-dimensional

vector space R_3.

The expression of $cos\alpha_{2,3}$ corresponding to the cosine of the angle between the second piecewise-linear straight line $\vec{z}_2$ and the predicting third vector straight line $\vec{Z}_3(1)$ on the base of the scalar product of 2 vectors, is represented in the form (Fig. 4):

$$cos\alpha_{2,3} = \frac{(\vec{z}_2 - \vec{z}_2^{k_2})\cdot(\vec{a}_4(1) - \vec{z}_2^{k_2})}{\left|\vec{z}_2 - \vec{z}_2^{k_2}\right|\left|\vec{a}_4(1) - \vec{z}_2^{k_2}\right|} \quad (63)$$

IV. Results Method of Numerical Calculation of 2-Component Economic-Mathematical Model and Definition of Predicting Vector Function with Regard to Unaccounted Factors Influence in 3-Dimensional Vector Space

Below we have given the numerical construction of a 2-component piecewise-linear economic mathematical model, and by means of the given model will determine the predicting function on the subsequent third small volume of the investigated economic process in 3-dimensional vector space [6-11,13-15]. Given a statistical table describing some economic process in the form of the points (vectors) set $\{\vec{a}_n\}$ in 3-dimensional vector space R_3. Represent the set of vectors $\{\vec{a}_n\}$ of statistical values in the form of adjacent 2-component piecewise-linear vector equation of the form Eq. (32):

$$\vec{z}_2 = \vec{z}_1^{k_1} + \mu_2(\vec{a}_3 - \vec{z}_1^{k_1}) \quad (64)$$

where $\vec{z}_1 = \vec{z}_1(z_{11}, z_{12}, z_{13})$ and $\vec{z}_2 = \vec{z}_2(z_{21}, z_{22}, z_{23})$ are the equations of the first and second piecewise-linear straight lines in 3-dimensional vector space; the vectors $\vec{a}_1(a_{11}, a_{12}, a_{13})$, $\vec{a}_2 = \vec{a}_2(a_{21}, a_{22}, a_{23})$ and $\vec{a}_3 = \vec{a}_3(a_{31}, a_{32}, a_{33})$ are given points (vectors) in 3-dimensional space of the form:

$$\vec{a}_1 = \vec{i}_1 + \vec{i}_2 + \vec{i}_3 \quad \vec{a}_2 = 3\vec{i}_1 + 2\vec{i}_2 + 4{,}5\vec{i}_3 \quad (65)$$

$$\vec{a}_3 = 6\vec{i}_1 + 4\vec{i}_2 + 7\vec{i}_3 \quad (66)$$

$\mu_1 \geq 0$ and $\mu_2 \geq 0$ are arbitrary parameter. Substituting Eq. (65) and (66) in Eq. (3264), the coordinate form of the vector equation of the first vector straight line will accept the form:

$$\vec{z}_1 = (1+2\mu_1)\vec{i}_1 + (1+\mu_1)\vec{i}_2 + (1+3{,}5\mu_1)\vec{i}_3 \quad (67)$$

As the intersection point of 2 straight lines $\vec{z}_1^{k_1}$ that should satisfy the conjugation condition $\vec{z}_1^{k_1} = \vec{z}_2^{k_1}$ may also not coincide with the point $\vec{a}_2$, then its appropriate value of the parameter μ_1 will be $\mu_1^{k_1} \geq 1$. In this connection, in numerical calculation, we accept the value of the parameter $\mu_1^{k_1}$ for the intersection point between piecewise-linear straight lines equal 1.5, i.e., $\mu_1^{k_1} = 1{,}5$. Then the value of the intersection point $\vec{z}_1^{k_1}$ Eq. (67) will equal:

$$\vec{z}_1^{k_1} = 4\vec{i}_1 + 2{,}5\vec{i}_2 + 6{,}25\vec{i}_3 \quad (68)$$

By Eq. (37) the equation of the second straight line in the vector form is expressed by the vector equation of the first piecewise-linear straight line $\vec{z}_1$ of the form Eq. (67) and the unaccounted parameter function $\omega_2(\lambda_2, \alpha_{1,2})$ in the form:

$$\vec{z}_2 = \vec{z}_1\{1 + A[1 + \omega_2(\lambda_2, \alpha_{1,2})]\} \quad (69)$$

Here the coefficient A and the unaccounted parameter function $\omega_2(\lambda_2, \alpha_{1,2})$ of the economic process will be of the form Eqs. (38)–(41) and (1143):

$$A = (\mu_1^{k_1} - \mu_1)\frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \quad \text{for } \mu_{1\geq}\ \mu_1^{k_1} = 1{,}5 \tag{70}$$

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = \lambda_2^{k_2}\cos\alpha_{1,2} \tag{71}$$

$$\lambda_2 = \frac{\mu_2^{k_2}}{\mu_1^{k_1} - \mu_1}\frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})}\frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|} \tag{72}$$

$$cos\alpha_{1,2} = \frac{(\vec{z}_1 - \vec{z}_1^{k_1})(\vec{z}_2 - \vec{z}_1^{k_1})}{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{z}_2 - \vec{z}_1^{k_1}|} \tag{73}$$

Here the parameter μ_2 corresponding to the points of the second piecewise-linear straight line is connected with the appropriate parameter μ_1 by Eq. (41). Here for the values $\mu_1 \geq \mu_1^{k_1} = 1{,}5$. In Eq. (73) the vector $\vec{z}_2$ is calculated by Eq. (65) for any value of μ_2 in the interval $0 \leq \mu_2 \leq 1$, and the vector $\vec{z}_1$ is of the form Eq. (64) for any value of $\mu_2 \geq \mu_1^{k_1}$. By calculating the value of the expression $Cos\alpha_{1,2}$ by Eq. (73), the value of $\vec{z}_1$ may be calculated for the value of $\vec{a}_3$ or for μ_2 that corresponds to the value of the second intersection point k_2, i.e., for $\mu \geq \mu_2^{k_2}$. Substituting the value of the parameter $\mu_1^{k_1} = 1{,}5$, and also Eq. (3466) in Eq. (41), set up a numerical relation between the parameters μ_2 and μ_1 in the form:

$$\mu_2 = 1{,}1927(\mu_1 - 1{,}5) \text{ for } \mu_1 \geq 1{,}5;\ 0 \leq \mu_2 \leq \mu_2^{k_2} > 1 \tag{74}$$

Thus, (74) is the numerical representation of mathematical relation between the parameters μ_1 and μ_2. Defining any value of $\mu_2 \geq 0$ by Eq. (74), it is easy to determine the appropriate value of the parameter μ_1. From (74) it will follow:

$$\mu_1 = 1{,}5 + 0{,}8384\mu_2 \tag{74a}$$

Calculate the values of the coefficient A, the unaccounted parameter function $\omega_2(\lambda_2, \alpha_{1,2})$ of economic process and $cos\alpha_{1,2}$. For that, substituting Eqs. (66)–(67) in Eq. (74), and also the numerical value of the parameter $\mu_1^{k_1} = 1{,}5$ in Eqs. (70)–(73), define the numerical values of A, $\omega_2(\lambda_2, \alpha_{1,2})$ and $Cos\alpha_{1,2}$ for $\mu_1 \geq\ 1{,}5$ in the form:

$$A = -(\mu_1 - 1{,}5)\frac{25{,}8751}{9{,}75 + 25{,}875 \cdot \mu_1} \tag{75}$$

$$\lambda_2 = 0{,}1208\frac{9{,}75 + 25{,}875\mu_1}{9{,}75 + 19{,}375\mu_1 0 - 17{,}25\mu_1^2}\sqrt{38{,}8125 - 51{,}25\mu_1 + 17{,}25\mu_1^2} \tag{76}$$

$$cos\alpha_{1,2} = 0.8495 \tag{77}$$

Numerical values of A and λ_2 for the second intersection point, i.e., for $\mu_1 = 3{,}1768$ calculated by Eqs. (75) and (76) will be equal to:

$A(3{,}1768) = -0{,}4719$, $\lambda_2 = (3{,}1768) = -0{,}7495$

Substituting Eqs. (67), (75)–(77) in Eq. (69), find the equation of the second vector straight line in the vector form depending on the vector function of the first piecewise-linear straight line and appropriate for the second linear straight line of the parameter $\mu_1 \geq\ 1{,}5$ in the form (Fig. 5):

$$\vec{z}_2 = \varphi_0(\mu_1)\cdot\vec{z}_1 = \varphi_0(\mu_1)\cdot[(1+2\mu_1)\vec{i}_1 + (1+\mu_1)\vec{i}_2 + (1+3{,}5\mu_1)\vec{i}_3]$$

for

$$\mu_1 \geq 1{,}5 \qquad (78)$$

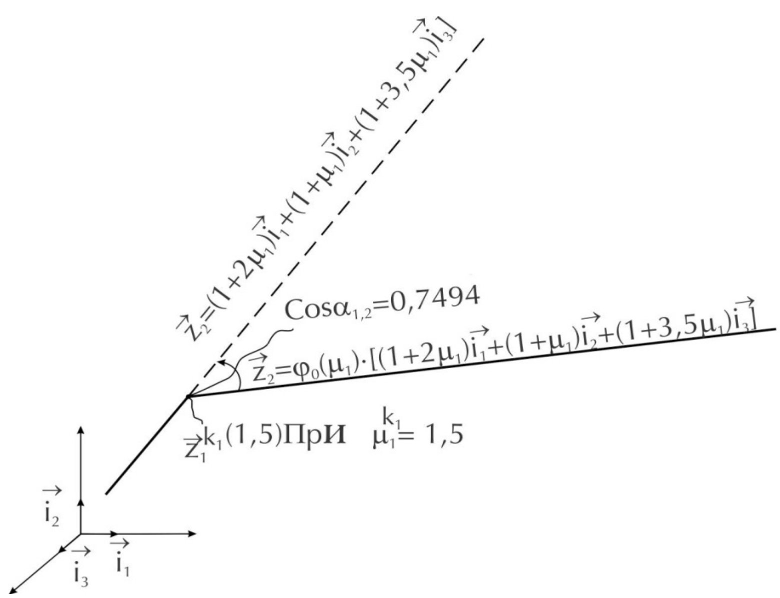
Here

$$\varphi_0(\mu_1) = 1-(\mu_1 - 1{,}5)\cdot\frac{25{,}8751}{9{,}75+25{,}875\cdot\mu_1}\left[1+ + 0{,}1026\frac{9{,}75+25{,}875\cdot\mu_1}{9{,}75+19{,}375\cdot\mu_1 - 17{,}25\mu_1^2}\sqrt{\begin{matrix}38{,}8125-51{,}75\cdot\mu_1+\\+17{,}25\mu_1^2\end{matrix}}\right] \qquad (79)$$

Numerical values $\varphi_0(\mu_1)$ at the second intersection point, i.e., for $\mu_1 = 3{,}1768$ will equal:

$\varphi_0(3{,}1768) = 0{,}8297$

Fig. 5. Numerical representation of 2-component piecewise-linear economic-mathematical model in 3-dimensional vector space R_3.



Now investigate the problem of prediction and control of economic process in the subsequent $\Delta V_3(x_1, x_2, x_3)$ volume of 3-dimensiona vector space with regard to unaccounted parameters factor that hold on preceding states of the process [6-11,13-15].Above for the case of 2-component piecewise-linear straight line it was numerically constructed the second vector straight line (78) depending on an arbitrary parameter μ_1 and unaccounted parameter influence space function $\omega_2(\lambda_2, \alpha_{1,2})$. On the other hand, for the 2-component case economic process a predicting vector function $\vec{Z}_3(1)$ with regard to the introduced unaccounted parameter influence predicting function $\Omega_3(\lambda_3, \alpha_{2,3})$ was suggested in the form:

$$\vec{Z}_3(1) = \vec{z}_1\{1 + A[1+\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) + \Omega_3(\lambda_3, \alpha_{2,3})]\} \qquad (80)$$

Here the coefficient A, the unaccounted parameter function $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$, and also the unaccounted parameter predicting function $\Omega_3(\lambda_3, \alpha_{2,3})$ are of the form Eqs. (53)–(58) define numerical values of these expressions. As the economic process predicting function $\vec{Z}_3(1)$ is the third piecewise-linear function, at first we define the value of the vector function $\vec{z}_2^{k_2}$ at the second intersection point k_2. The parameter μ_2 acting on the segment of the second piecewise-linear straight line changes in the interval $0 \le \mu_2 \le \mu_2^{k_2} \ge 1$. Here the value of the parameter $\mu_2^{k_2}$ belongs to the intersection point between the second and third straight lines. According to approximation of statistical points, this point should be defined. Therefore, giving the value of the parameter $\mu_2^{k_2}$ at the second intersection point k_2, define from Eq. (41) the appropriate value of the parameter $\mu_1^{k_2}$, in the form:

$$\mu_1^{k_2} = \mu_1^{k_1} + \mu_2^{k_2} \frac{(\vec{a}_3 - \vec{z}_1^{k_1})^2}{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)} \quad (81)$$

For conducting numerical calculation we accept $\mu_2^{k_2} = 2$. For the value of the parameter $\mu_2^{k_2} = 2$, we define the appropriate numerical value of the parameter μ_1, that will be denoted by $\mu_1^{k_2}$, from Eq. (81) or Eq. (74). It will equal:

$$\mu_1^{k_2} = 3{,}1768 \quad (82)$$

Thus, we established the range of the parameter μ_1 corresponding to the change of the parameter μ_2 of the segment of the second piecewise-linear straight line, in the form:

$$1{,}5 \le \mu_1 \le 3{,}1768 \text{ for } 0 \le \mu_2 \le \mu_2^{k_2} = 2 \quad (83)$$

Though Eq. (81) is valid for the values of the parameter $\mu_2 \ge 2$ as well. In this case, the value of the prediction function $\vec{Z}_3^{k_2}(1)$ at the intersection point k_2, i.e., for $\mu_3 = 0$, $\mu_2 = 2$, $\mu_1^{k_2} = 3{,}1768$ coincides with the value of the function of the second piecewise-linear straight line:

$$\vec{Z}_3^{k_2}(1) = \vec{z}_2^{k_2} \quad (84)$$

Note that at the intersection point k_2, i.e., for $\mu_2^{k_2} = 2$, $\mu_3^{k_2} = 0$ the unaccounted parameters influence predicting function $\Omega_3(\lambda_3, \alpha_{23}) = 0$.But the function $\vec{z}_2$ has the form (78). Therefore, it suffices to substitute to Eq. (78) the value of the parameter $\mu_1^{k_2} = 3{,}1768$ that will be defined both as the value of the predicting function $\vec{Z}_3^{k_2}(1)$ at the initial point $\mu_2 = 2$ $\mu_3 = 0$ of the third vector straight line and the value of the point $\vec{z}_2^{k_2}$ at the final point of the second piecewise-linear straight line at the point к2, in the form:

$$Z_3^{k_2}(1)\Big|_{\mu_1=3{,}1768} = 6{,}1013\vec{i}_1 + 3{,}4655\vec{i}_2 + 10{,}055\vec{i}_3$$

for $\mu_2 = 2$, $\mu_1^{k_2} = 3{,}1768$, $\mu_3 = 0$ (85)

Calculate the point $\vec{a}_4(1)$. For that give in an arbitrary form 1 of the coordinates of the vector $\vec{a}_4(1)$, for instance, the coordinate $a_{41}(1)$, and by Eq. (61) calculate the remaining coordinates of the vector $\vec{a}_4(1)$. Furthermore, $a_{41}(1)$ is given so that $a_{41}(1)$ were greater than the coordinates $z_{21}^{k_2} = 5{,}8411$. Therefore accept the value $a_{41}(1)$ =6,5. In this case, substituting Eqs. (66) and (85) in Eq. (61), define the vector $\vec{a}_4(1)$ in the coordinate form depending on an arbitrarily given value of $a_{41}(1)$ in the form:

$$\vec{a}_4(1) = a_{41}\vec{i}_1 + (1{,}3707 + \frac{1}{3}a_{41})\vec{i}_2 + (-3{,}5163 + 2{,}25a_{41})\vec{i}_3 \quad (86)$$

For the value $a_{41}(1)$ =six,5, the vector accepts the form $\vec{a}_4(1)$:

$$\vec{a}_4(1) = 6{,}5\vec{i}_1 + 3{,}5374\vec{i}_2 + 11{,}1087\vec{i}_3 \quad (87)$$

For numerical definition of the coefficient A the unaccounted parameter function $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ and also the unaccounted parameter predicting function $\Omega_3(\lambda_3, \alpha_{2,3})$ allowing for Eqs. (66)–(68), (74), (79), (41) and (85) conduct the following calculations:

1) $\left|\vec{z}_1^{k_1} - \vec{a}_1\right| = \left|4\vec{i}_1 + 2{,}5\vec{i}_2 + 6{,}25\vec{i}_3 - \vec{i}_1 - \vec{i}_2 - \vec{i}_3\right| = 6{,}23$ (88)

2) $\left|\vec{a}_2 - \vec{a}_1\right| = \left|2\vec{i}_1 + \vec{i}_2 + 3{,}5\vec{i}_3\right| = 4{,}1533$ (89)

3) $\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1) = \left[(1+2\mu_1)\vec{i}_1 + (1+\mu_1)\vec{i}_2 + (1+3{,}5\mu_1)\vec{i}_3\right](3\vec{i}_1 +$
$+1{,}5\vec{i}_2 + 5{,}25\vec{i}_3) = 9.75 + 25{,}875\mu_1 = A_1(\mu_1)$ (90)

4) $\left(\vec{z}_2 - \vec{z}_2^{k_2}\right) = \{\varphi_0(\mu_1)\left|(1+2\mu_1)\vec{i}_1 + (1+\mu_1)\vec{i}_2 + (1+3{,}5\mu_1)\vec{i}_3\right| -$
$-5{,}8411\vec{i}_1 - 3{,}3177\vec{i}_2 - 9{,}6262\vec{i}_3\} =$
$= \left[\varphi_0(1+2\mu_1) - 5{,}8411\right]\vec{i}_1 + \left[\varphi_0(1+\mu_1) - 3{,}3177\right]\vec{i}_2 +$
$+ \left[\varphi_0(1+3{,}5\mu_1) - 9{,}6262\right]\vec{i}_3$ (91)

5) $\left|\vec{z}_2 - \vec{z}_2^{k_2}\right| = \left|\ \varphi_0(\mu_1)\left[(1+2\mu_1)\vec{i}_1 + (1+\mu_1)\vec{i}_2 + (1+3{,}5\mu_1)\vec{i}_3\right] -\right.$
$\left. -5{,}8411\vec{i}_1 - 3{,}3177\vec{i}_2 - 9{,}6262\vec{i}_3\ \right| =$

$$= \sqrt{\begin{matrix}\left[\varphi_0(1+2\mu_1) - 5{,}8411\right]^2 + \left[\varphi_0(1+\mu_1) - 3{,}3177\right]^2 + \\ + \left[\varphi_0(1+3{,}5\mu_1) - 9{,}6262\right]^2\end{matrix}} = A_2(\mu_1) \quad (92)$$

6) $\vec{z}_1(\vec{z}_2 - \vec{z}_2^{k_2}) = \varphi_0(\mu_1)[(1+2\mu_1)^2 + (1+\mu_1)^2 + (1+3{,}5\mu_1)^2] -$

$-[18{,}785 + 48{,}6916\mu_1] = A_4(\mu_1)$ (93)

7) $\left|\vec{a}_4(1) - \vec{z}_2^{k_2}\right| =$
$= \left| a_{41}\vec{i}_1 + (1{,}3707 + \frac{1}{3}a_{41}\vec{i}_2 + (-3{,}5163 + 2{,}25a_{41})\vec{i}_3 - \right.$
$\left. -5{,}8411\vec{i}_1 - 3{,}3177\vec{i}_2 - 9{,}6262\vec{i}_3 \right| =$

$$= \sqrt{\begin{matrix}(a_{41}(1) - 5{,}8411)^2 + (-1{,}947 + \frac{1}{3}a_{41}(1))^2 + \\ + (-13{,}1425 + 2{,}25a_{41}(1))^2\end{matrix}} = A_3(\mu_1) \quad (94)$$

8) $[\vec{a}_4(1) - \vec{z}_2^{k_2}]^2 =$
$= (a_{41}(1) - 5{,}8411)^2 + (-1{,}947 + \frac{1}{3}a_{41}(1))^2 +$
$+ (-13{,}1425 + 2{,}25a_{41}(1))^2$ (95)

9) $(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_4(1) - \vec{z}_2^{k_2}) =$
$= 2(a_{41}(1) - 6{,}1013) + 1{,}5(-1{,}947 + \frac{1}{3}a_{41}a_{41}(1)) +$
$+ 0{,}75(-13{,}1425 + 2{,}25a_{41}(1))$ (96)

Substituting the values $a_{41}(1)$ =6.5 and Eq. (86) in Eqs. (86)–(98), we have:

$\left|\vec{a}_4(1)-\vec{z}_2^{k_2}\right|$ =1,9929 (97)

$[\vec{a}_4(1)-\vec{z}_2^{k_2}]^2$ =3,9715 (98)

$(\vec{a}_3-\vec{z}_1^{k_1})(\vec{a}_4(1)-\vec{z}_2^{k_2})$ =2,5532 (99)

Now set up numerical relation between the parameters μ_3 and μ_1. For that, substituting Eqs. (97)–(99), and taking into account the numerical values $a_{41}(1)=6{,}5$ and $\mu_2^{k_2}=2$, the relation Eq. (58) between the parameters will be of the form:

$$\mu_3=(\mu_2-2)\frac{2{,}5532}{3{,}9715} \text{ for } \mu_2\geq 2,\ \mu_3\geq 0 \quad \text{or} \quad \mu_3=1{,}6429(\mu_2-2) \qquad (100)$$

Substituting the numerical dependence between the parameters μ_2 and μ_1 in the form Eq. (74) in (100), set up dependence of the parameter μ_3 on the parameter μ_1 in the form:

$$\mu_3=0{,}7668(\mu_1-3{,}1768) \text{ for } \mu_1\geq 3{,}1768 \qquad (100a)$$

or

$$\mu_1=1{,}3041\cdot\mu_3+3{,}1768 \qquad \text{for } \mu_3\geq 0$$

Now, substituting Eqs. (88), (89), (90), (92), (93), (94), and (100) in Eq. (57), define the unaccounted factors predicting parameter λ_3 in the form:

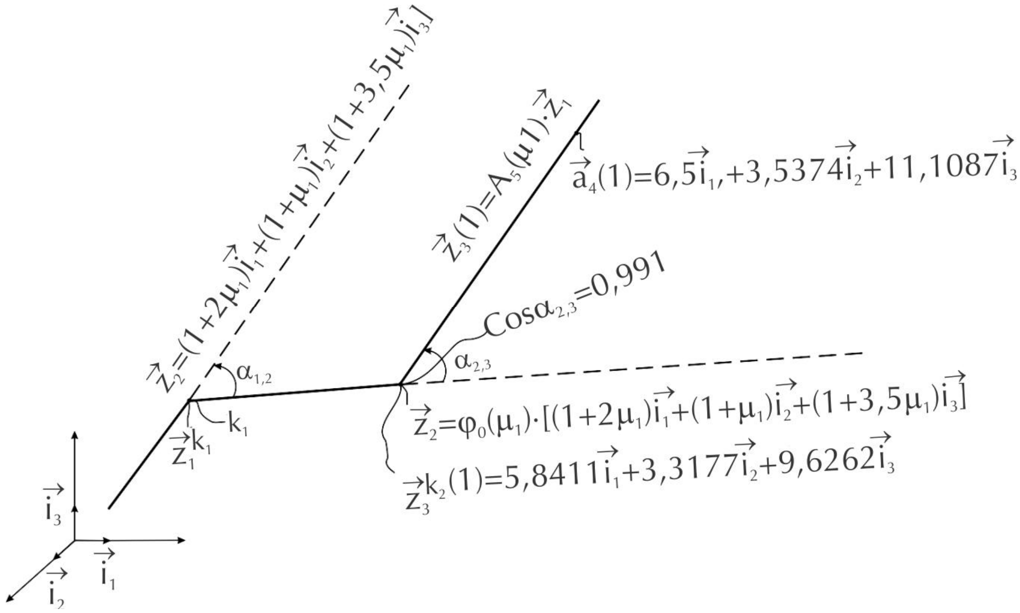
$$\lambda_3=-0{,}0296\frac{\mu_1-3{,}1768}{\mu_1-1{,}5}\cdot\frac{A_1(\mu_1)A_2(\mu_1)A_3(\mu_1)}{A_4(\mu_1)} \qquad \text{for} \qquad \mu_1\geq 3{,}1768 \qquad (101)$$

where $\varphi_0(\mu_1)$ is of the form Eq. (79).

Now, by Eq. (63), calculate the cosine of the angle $cos\alpha_{23}$ between the economic process predicting vector function $\vec{Z}_3(1)$ and the second piecewise-linear vector-function $\vec{z}_2(\mu_2)$ in the form (Fig. 6.):

$$cos\alpha_{2,3}=\frac{(\vec{a}_4(1)-\vec{z}_2^{k_2})(\vec{z}_2(\mu_2)-\vec{z}_2^{k_2})}{\left|\vec{a}_4(1)-\vec{z}_2^{k_2}\right|\left|\vec{z}_2(\mu_2)-\vec{z}_2^{k_2}\right|} \qquad (102)$$

Fig. 6. Numerical construction of predicting vector function $\vec{Z}_3(\beta)$ on the base of 2-component economic-mathematical model in 3-dimensional vector space R_3.



Taking into account Eqs. (91)–(95), expression of $cos\alpha_{2,3}$ takes the form:

$$cos\alpha_{2,3}=$$

$$=\frac{(6{,}7263\mu_1+2{,}3611)\varphi_0(\mu_1)-18{,}8484}{1{,}6372\sqrt{\begin{array}{l}[\varphi_0(1+2\mu_1)-5{,}8411]^2+[\varphi_0(1+\mu_1)-3{,}3177]^2+\\+[\varphi_0(1+3{,}5\mu_1)-9{,}6262]^2\end{array}}}$$

For $\mu_1=5$ the numerical value of $cos\alpha_{2,3}$ will be:

$$cos\alpha_{2,3}=0{,}9448 \qquad (103)$$

From Eq. (2456) calculate $\Omega_3(\lambda_3,\alpha_{2,3})$. For that substitute Eqs. (90)–(100), (92), (6294), (100), (103) in Eq. (56), and calculate $\Omega_3(\lambda_3,\alpha_{2,3})$:

$$\Omega_3(\lambda_3,\alpha_{2,3})=0{,}028\frac{\mu_1-3{,}1768}{1{,}5-\mu_1}\cdot\frac{A_1(\mu_1)A_2(\mu_1)A_3(\mu_1)}{A_4},$$

For

$$\mu_1\geq 3{,}1768 \qquad (104)$$

Now calculate the unaccounted parameter function $\omega_2(\lambda_2^{k_2},\alpha_{1,2})$ belonging to the second piecewise-linear straight line, and take into account the character of relation between the parameters μ_2 and μ_1 given in the form Eq. (74):

$$\mu_2=1{,}1927(\mu_1-1{,}5) \text{ for } \mu_1\geq 1{,}5,\ 0\leq\mu_2\leq\mu_2^{k_2}>1 \qquad (105)$$

Hence:

$$\mu_1=1{,}5+0{,}8384\mu_2 \qquad (106)$$

For $\mu_2=\mu_2^{k_2}$ from Eq. (106):

$$\mu_1^{k_2}=1{,}5+0{,}8384\mu_2^{k_2} \qquad (107)$$

For the considered example, for the second intersection point k_2 the value of the parameter $\mu_2^{k_2}$ earlier was accepted to be equal to 2, i.e., $\mu_2^{k_2}=2$. In this case, the appropriate numerical value of the parameter $\mu_1^{k_2}$ by Eq. (107) will equal:

$\mu_1^{k_2}$ =3,1768 (108)

Now carry out appropriate calculations by Eq. (53) for defining $\omega_2(\lambda_2^{k_2},\alpha_{1,2})$, and calculate the vector $\vec{z}_1(\mu_1)$ in it for the value of the parameter $\mu_1=\mu_1^{k_2}=3{,}1768$. Taking into account $\mu_1^{k_1}=1{,}5$, $\mu_2^{k_2}=2$, $\mu_1^{k_2}$ =3,1768, $cos\alpha_{1,2}=0{,}8495$, and also Eqs. (45), (56)–(58), define the numerical value of $\omega_2(\lambda_2^{k_2},\alpha_{1,2})$ in the form:

$$\omega_2(\lambda_2^{k_2},\alpha_{1,2})=\lambda_2^{k_2}\cdot cos\alpha_{12}=-0{,}635 \quad (109)$$

Substituting Eqs. (88)–(90) in Eq. (55), express the coefficient A by the parameter $\mu_1\geq\mu_1^{k_2}=3{,}1768$ in the form:

$$A=-25{,}875\frac{\mu_1-1{,}5}{A_1(\mu_1)} \text{ for } \mu_1\geq\mu_1^{k_2}=3{,}1768 \quad (109a)$$

where

$$A_1(\mu_1)=9{,}75+25{,}875\mu_1$$

Substituting the numerical values of the coefficient A Eq. (109a), the unaccounted parameter influence function $\omega_2(\lambda_2^{k_2},\alpha_{1,2})$ Eq. (109) and also the unaccounted parameter influence predicting function $\Omega_3(\lambda_3,\alpha_{2,3})$ Eq. (104) in Eq. (52), for the case of 2-component piecewise-linear straight line find the form of the economic process predicting vector function $\vec{Z}_3(1)$ in 3-dimensional vector space in the form (Fig. 6) [4–6]:

$$\vec{Z}_3(1)=\vec{z}_1\left\{1-9{,}4444\frac{\mu_1-1{,}5}{A_1(\mu_1)}\cdot[1-\right.$$

$$\left.-0{,}0767\frac{\mu_1-3{,}1768}{\mu_1-1{,}5}\cdot\frac{A_1(\mu_1)A_2(\mu_1)A_3(\mu_1)}{A_4(\mu_1)}\right\} \text{ for } \mu_1\geq 3{,}1768, \quad (110)$$

where

$$\vec{z}_1=(1+2\mu_1)\vec{i}_1+(1+\mu_1)\vec{i}_2+(1+3{,}5\mu_1)\vec{i}_3 \quad (111)$$

$$A_1(\mu_1)=9.75+25{,}875\mu_1 \quad (112)$$

$$A_2(\mu_1)=\sqrt{\begin{matrix}[\varphi_0(1+2\mu_1)-5{,}8411]^2+[\varphi_0(1+\mu_1)-3{,}3177]^2+\\+[\varphi_0(1+3{,}5\mu_1)-9{,}6262]^2\end{matrix}} \quad (113)$$

$$A_3(\mu_1)=\sqrt{\begin{matrix}(a_{41}-5{,}8411)^2+(-1{,}947+\frac{1}{3}a_{41})^2+\\+(-13{,}1425+2{,}25a_{41})^2\end{matrix}} \quad (114)$$

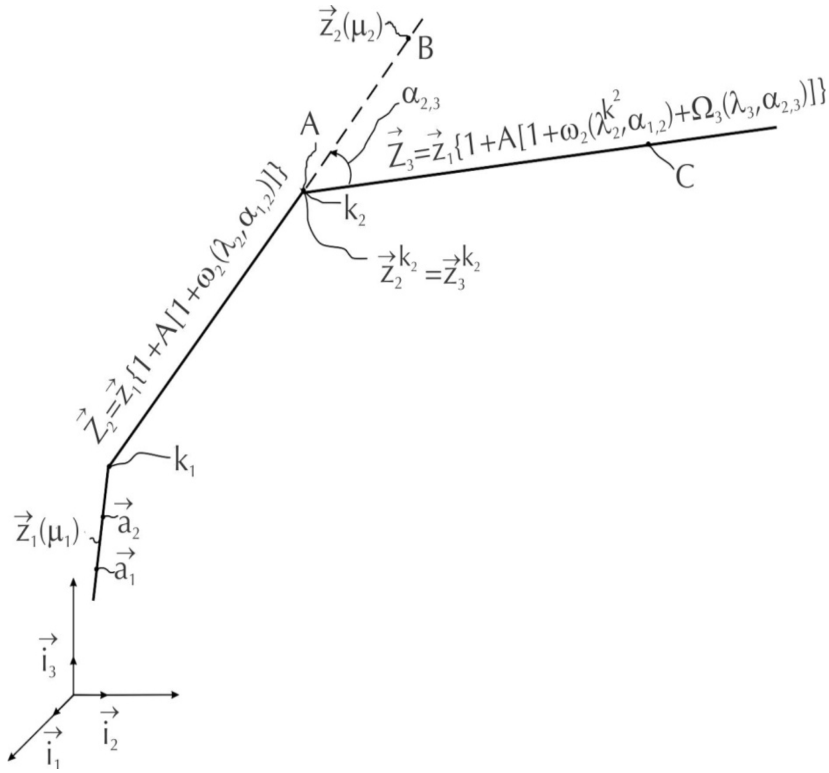
$$A_4(\mu_1)=\varphi_0(\mu_1)[(1+2\mu_1)^2+(1+\mu_1)^2+(1+3{,}5\mu_1)^2]-$$
$$-[18{,}785+48{,}6916\mu_1] \quad (115)$$

$$\varphi_0(\mu_1)=1-(\mu_1\text{ -}1{,}5)\cdot\frac{25{,}8751}{9{,}75+25{,}875\cdot\mu_1}\left[1+\right.$$

$$\left.+0{,}1026\frac{9{,}75+25{,}875\cdot\mu_1}{9{,}75+19{,}375\cdot\mu_1-17{,}25\mu_1^2}\sqrt{\begin{matrix}38{,}8125-51{,}75\cdot\mu_1+\\+17{,}25\mu_1^2\end{matrix}}\right] \quad (116)$$

Eq. (78) is written in the compact form as follows (Fig. 7):

Fig. 7. Compact form of representation of numerical expression of the predicting vector function $\vec{Z}_3(\beta)$ constructed on the base of 2-component model in 3-dimensional vector space R_3.



$$\vec{Z}_3(1) = A_5(\mu_1)\cdot \vec{z}_1 \quad (117)$$

where

$$A_5(\mu_1) = 1 - 9{,}4444\frac{\mu_1 - 1{,}5}{A_1(\mu_1)}\cdot[1-$$

$$-0{,}0767\frac{\mu_1 - 3{,}1768}{\mu_1 - 1{,}5}\cdot\frac{A_1(\mu_1)A_2(\mu_1)A_3(\mu_1)}{A_4(\mu_1)} \quad \text{for} \quad \mu_1 \geq 3{,}1768 \quad (118)$$

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