

## Portfolio Optimization of Financial Services Stocks in the Nigerian Stock Exchange

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### Abstract

This paper attempts to maximize the expected return and minimize the variance of the portfolio by using Markowitz's portfolio selection model and a three-objective linear programming model to allocate different percentage of weight to different assets to obtain an optimal/feasible portfolio of the financial sector of the Nigerian stock exchange (NSE). An equally weighted portfolio was constructed using the daily closing prices from the financial services sector of NSE. The mean and the standard deviation of the data from the sector of the market served as our constraints in the three-objective model used. Additionally, three portfolios were constructed with the aims of maximizing the returns and minimizing the standard deviation (variance) and maximizing the Sharpe ratio. With the result from the simulation and analysis, these portfolios were compared alongside with the original equally weighted portfolio and the result of the comparison form the basis for our recommendations provided to the investors and market practitioners in the sector of NSE.

**Keywords:** Portfolio optimization; Weight allocation; Diversification of assets; Sharpe ratio; and Correlation matrix

### 1. Introduction

In early 1950's, Harry Markowitz designed a financial model otherwise called mean-variance portfolio optimization. This method was designed such that it will help the investors know which asset that will be selected in a portfolio, how the selection will be done and also the weight of each asset in the portfolio. In the paper titled Portfolio selection (1952), Markowitz's outlined the importance of diversification of portfolios. However, research has shown that the Markowitz mean-variance has some weaknesses and a number of limitations. As a matter of fact, the limitations have taken the centre stage of research. Researchers like: Fuerst (2008), Norton (2009), Ceria and Stubbs (2006), Goldfarb and Iyengar (2003), Jorion (1992), Konno and Suzuki (1995), Michaud (1989a) (1989b), Bowen (1984), Ravipiti (2012) etc. discussed the weaknesses, limitations and assumptions in their works.

Since the discovering of the Markowitz's MV limitations and weaknesses, a lot of researchers have been working on the model to improve and develop it in different directions. Authors like Jobson, Korkie and Ratti (1979), Jobson and Korkie (1980), Frost and Savarino (1988), Jorion (1992), Michaud (1998), (1989b), Polson and Tew (2000), etc. worked on the estimation error.

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Others like Britten-Jones (2002), Kandel and Stambaugh (1996), Zellner and Chetty (1965), Klein and Bawa (1976) and Brown (1978) worked on the Markowitz's model by using Bayesian approach and predictive probability to improve and develop the model in various ways. Huang (2008) and Markowitz (1993) tried to develop the model to Mean-semi variance. Authors like Galluccio et al. (1998), Laloux et al. (1999), (2008), Bongini et al. (2002), Pafka and Kondor (2002), (2003), Potters et al. (2005), Lindberg (2009) and others brought in Random matrix theory (RMT), which was first proposed and introduced by Wigner (1951) and Laloux and Plerou introduced RMT in financial markets, to improve Markowitz's portfolio optimisation.

In this paper, we aim to optimize a portfolio containing stocks from the financial services sector of the Nigerian stock Exchange (NSE) using Markowitz's portfolio selection model and a three-objective linear programming model to allocate different percentage of weight to different assets to obtain an optimal/feasible portfolio, diversification of assets, and later we brought in cross - correlation of the individual stock of the sector to show the relationship between any two assets chosen in the correlation matrix. The rest of the paper is organised as follows. In section 2, we describe the nature of the empirical data used in the analysis. In section 3, we present the methodologies, theoretical background on mean-variance optimization; expected return and risk of the portfolio of the assets, constrain objective programme and cross - correlation of assets. Section 4, shows and discusses the main empirical result. Finally, section 5 concludes the paper.

## 2. Data

We obtained our data from NSE, which is made up of eleven (11) sectors, but our analysis is on the financial services. The financial services are about 57 assets from the time our analysis started. But we have to bring them down to 24 assets only in the course of our study. This development was necessary because some of assets were delisted from NSE due to some banks merging together and some bigger ones acquiring smaller one after the global melt down in order to meet up with the new capital base for the financial institutions operating in the country as ordered by the Central Bank of Nigeria (CBN). Also, some of the company under this sector did not trade more regularly within the time interval of our analysis and therefore, was removed. The data set used is the daily closing price of the stock data listed in the financial services of NSE. We have 1485 daily closing prices running from 3rd August 2009 to 4th August 2015, excluding weekends and public holidays in Nigeria (Nationwide). These stock price data were converted into 1484 logarithmic returns and was used in our analysis.

Let  $P_i(t)$  be the closing price of the index on day  $(t)$  of stock  $i$  and define the natural logarithmic returns of the index (i.e. the log-difference of  $P_i(t+1)$  and  $P_i(t)$ ) as

$$r_i(t) = \ln P_i(t+1) - \ln P_i(t) \quad (1)$$

Where  $r_i(t)$  has 1484 observation? Before establishing the portfolio selection process, we compute the mean return and standard deviation of each stock  $i$ .

The table below shows the mean and standard variation of the individual stocks.

## 3. Theoretical background and methodology

### 3.1 Expected return of a portfolio.

The portfolio of  $n$  assets has each  $i^{th}$  asset delivers a return of  $r_i(t)$  at the time  $t$ . Each  $r_i(t)$  has its mean and variance which is denoted as  $\mu_i(t)$  and  $\sigma_i^2(t)$  respectively. The money invested in the  $i^{th}$  assets is regarded as weight of the asset ( $w_i$ ) (which is less than 1 and sometimes a negative number is allowed if there is a short selling of any asset). Therefore, the summation of the individual weights of the assets that form the portfolio is 1, thus,  $\sum_{i=1}^n w_i = 1$  and it is obvious to see that

$$\mu = E[r] = E \left[ \sum_{i=1}^n w_i r_i \right]$$

Therefore,

$$\mu = \sum_{i=1}^n w_i \mu_i \tag{2}$$

and

$$\sigma^2(r) = E[(r - \mu)^2] = E \left[ \left( \sum_{i=1}^n w_i (r_i - \mu_i) \right)^2 \right]$$

Which implies that

$$\sigma^2(r) = \sum_{i,j=1}^n w_i w_j \sigma_{i,j} \tag{3}$$

Which is written as  $\sigma^2(r) = w^T V w$ , where  $w = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$  and  $V = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix}$  are called the weight vector and covariance matrix respectively.

Let's recall that the correlation between any two assets is

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{4}$$

Where  $\sigma_i, \sigma_j$  are the standard deviation of  $i$  and  $j$  respectively, while  $\rho_{ij}$  is the coefficient of correlation of  $i$  and  $j$ , for  $i, j = 1, 2, \dots, n$ . The coefficient of correlation plays a great role in the portfolio diversification, if well managed; the coefficient of correlation will reduce the risk to a bearable level. In other words, the risk of a portfolio decreases as the coefficient of correlation of the assets moves from positive to negative.

**Table 1**

	ACCESS	AUDO	CONTINSURE	CORNERST	CUSTODIANS	DIAMONDBANK	FBNH	FICMB	FIDELITYBK	GUARANTY	IMANSARD	INEM	NIGERINS	PRESTIGE	ROYALEX	SKYEBANK	STERLINGBANK	TRANSFCORP	UAC-PROP	UBA	UBN	WAPIC	WEMABANK	ZENITHBANK
Average	-0.0218%	-0.0229%	-0.0284%	-0.0334%	0.0253%	-0.0510%	-0.0606%	-0.0640%	-0.0409%	0.0335%	-0.0140%	0.4518%	-0.0736%	-0.1566%	-0.0447%	0.1844%	0.0235%	0.0907%	-0.0235%	0.0081%	-0.1663%	0.0047%	0.0182%	0.0588%
Variance	0.0686%	0.1036%	0.0878%	0.0338%	0.0971%	0.0785%	0.0546%	0.0692%	0.0779%	0.0577%	0.0859%	3.1963%	0.0416%	0.0824%	0.0677%	1.1029%	0.0960%	0.1124%	0.0868%	0.2022%	0.5122%	0.2607%	0.2148%	0.0659%
Standard D	2.6188%	3.2192%	2.9637%	1.8390%	3.1164%	2.8021%	2.3362%	2.6303%	2.7915%	2.4023%	2.9312%	17.8781%	2.0384%	2.8703%	2.6021%	10.5018%	3.0976%	3.3522%	2.8906%	4.4964%	7.1569%	5.0997%	4.6348%	2.4603%

Table showing individual assets return, variance and risks from financial sector of NSE. It is understood that the intention of every investor is to make as much gain as possible; therefore, it will be his wish to select the optimal portfolio which will maximize his expected return.

This can be expressed in a mathematical form thus;

$$f(w) = \text{Max } E[r]$$

$$\text{Subject to. } \sum_{i=1}^n w_i = 1 \tag{5}$$

$$w_i \geq 0, i = 1, 2, \dots, n$$

This implies that all the funds will be invested in the  $n$  assets and in the course of our analysis there will be no short selling.

An investor wishes to build a feasible portfolio  $w^*$ ; this feasible portfolio becomes the efficient one if it satisfies the following condition with at least one strict inequality;

1.  $\mu(w) \leq \mu(w^*)$
2.  $\sigma(w) \geq \sigma(w^*)$
3.  $SR(w) \leq SR(w^*)$

Where  $\mu(w)$ ,  $\sigma(w)$  and  $SR(w)$  are the expected return, risk and the sharpe ratio of the portfolio  $w = (w_1, w_2, \dots, w_n)$ .

This gives us a model of three - objective programming problem which shows that the expected return and the sharpe ratio will be maximised and the variance will be minimised.

Thus the model becomes,

$$f(w_1, w_2, \dots, w_n) \begin{cases} \text{Max } \mu(w) \\ \text{Min } \sigma(w) \\ \text{Max } SR(w) \end{cases} \tag{6}$$

Subject to

$$\sum_{i=1}^n w_i = 1$$

With  $w_i \geq 0, i = 1, 2, \dots, n$ .

The mean-variance criterion is also equivalent to the expected utility approach for any risk-averse utility function, when all returns are normal random variables. Since the probability distribution is defined on mean and standard deviation, it implies that expected utility is a function of mean and standard deviation. When the utility is risk averse, therefore,

$$E[U(w)] = f(\mu, \sigma) \tag{7}$$

$$\text{with } \frac{\partial f}{\partial \mu} > 0 \text{ and } \frac{\partial f}{\partial \sigma} < 0.$$

Where  $U$  is the utility function,  $\mu$  is the mean and  $\sigma$  is the standard deviation To solve the maximization function problem of a portfolio is therefore, a linear combination of assets that are normal random variables with respect to all feasible combinations. Our task now is to find  $w^*$  that will maximize  $f(\mu, \sigma)$  with respect to all feasible combination

#### 4. Empirical results and its analysis

Our main aim is to maximise the expected return and minimise the variance of the expected return of the portfolio containing assets from the financial services using the daily closing prices of the assets from 3<sup>rd</sup> of August 2009 to 4<sup>th</sup> of August 2015. This becomes 1484 days when all weekends and public holidays in Nigeria are excluded.

##### 4.1 Portfolio1 Equally weighted Portfolio

We first constructed a portfolio that is equally weighted using the daily closing prices of the market, we got a portfolio which the return is 0.00162% and the standard deviation is 1.28% (see Table 2 and 3). Though, the standard deviation of the portfolio seems to be better than what we have from the market (see Table 1 and 4), but the return is very poor. In Table 4 and figure 1, we can see that the single asset with the least risk is CORNERST, which is 1.84% but unfortunately, with a return that is very poor. Now our objective is to maximise the portfolio's return with a portfolio standard deviation which should be less than or equal to the least risk, (in other words we want construct a portfolio that the standard deviation will be less than or equal to that of CORNERST but the return will be above its return).

##### 4.1 Portfolio 2: Maximization of the return

Therefore, we apply

$$\text{maximize } f(w_1, w_2, \dots, w_n) = \sum_{i=1}^n w_i \mu_i - \mu_p \quad (8)$$

Subject:

$$g_1(w_1, w_2, \dots, w_n) = \frac{1}{n-1} \sum_{i=1}^n w_i w_j \sigma_{i,j} \leq 1.84\%$$

$$g_2(w_1, w_2, \dots, w_n) = \sum_{i=1}^n w_i - 1 = 0$$

Where  $w_i$  is the weight of individual assets,  $n$  is the number of the observations. After the simulation, the weights were distributed among the assets but assets like UBA, UBN, Diamond bank, ACCESS, FBNH, Fidelity bank, FCMB etc. were allocated with 0% of the weight while assets like Transcorp, Guaranty trust bank and Custody were given more percentage of the weight (see Table 4).

Table 2

	Portfolios			
	Equal Wt	Max Return	Min St Dev	Max SR
	None	at $\sigma \leq$	at $\mu =$	None
Value of Constr	N/a	1.840%	0.450%	N/a
ACCESS	4.1666%	0.0000%	0.0000%	0.0000%
AIICO	4.1666%	0.0000%	0.0000%	0.0000%
CONTINSURE	4.1666%	0.0000%	0.0000%	0.0000%
CORNERST	4.1666%	0.0000%	0.0000%	0.0000%
CUSTODYINS	4.1666%	11.9229%	0.0000%	10.3272%
DIAMONDBNK	4.1666%	0.0000%	0.0000%	0.0000%
FBNH	4.1666%	0.0000%	0.0000%	0.0000%
FCMB	4.1666%	0.0000%	0.0000%	0.0000%
FIDELITYBK	4.1666%	0.0000%	0.0000%	0.0000%
GUARANTY	4.1666%	27.2599%	0.0000%	26.4968%
MANSARD	4.1666%	0.0000%	0.0000%	0.0000%
NEM	4.1666%	5.1779%	100.0000%	6.3839%
NIGERINS	4.1666%	0.0000%	0.0000%	0.0000%
PRESTIGE	4.1666%	0.0000%	0.0000%	0.0000%
ROYALEX	4.1666%	0.0000%	0.0000%	0.0000%
SKYEBANK	4.1666%	5.6436%	0.0000%	6.7855%
STERLNBANK	4.1666%	8.5136%	0.0000%	6.7090%
TRANSCORP	4.1666%	36.2113%	0.0000%	40.8552%
UAC-PROP	4.1666%	0.0000%	0.0000%	0.0000%
UBA	4.1666%	0.0000%	0.0000%	0.0000%
UBN	4.1666%	0.0000%	0.0000%	0.0000%
WAPIC	4.1666%	2.0102%	0.0000%	0.7009%
WEMABANK	4.1666%	2.9119%	0.0000%	1.7415%
ZENITHBANK	4.1666%	0.3488%	0.0000%	0.0000%
$\Sigma w$	100.00%	100.00%	100.00%	100.00%
$\mu_p$	0.00162%	0.0839%	0.450%	0.0946%
$\sigma_p$	1.281%	1.840%	17.89%	2.060%
$\mu/\sigma$	0.126%	4.56%	2.52%	4.59%

Table of our four different portfolios constructed.

Though in this new portfolio, we got a standard deviation that is greater than that of the portfolio with equal weighted assets, but the return is very encouraging. The return is about 52 times of the return of the said portfolio (see Table 4). Again, if we look at the return of the asset with the least standard deviation (CORNERST with  $\sigma = 1.84\%$ , see Table 3), you will notice that it cannot be compared to our new return. Finally, if we look at the Sharpe ration (SR) of the portfolios, SR of the equal weighted portfolio and our new portfolio are 0.12% and 4.56% respectively (see Table 4) and the stock with the least standard deviation has its SR to be 1.81% (Table 2), this shows that 4.56% is best among all.

### 4.3 Portfolio 3: Minimization of Standard Deviation

#### 4.4

In this case, we want to minimise the standard deviation of the single asset with maximum SD (NEM) which is 17.88% (Table 1), to see if we will get a lower SD and an improved return (which may not necessarily be equal to the return of the said asset). Therefore, we apply

$$\text{minimize } f(w_1, w_2, \dots, w_n) = \frac{1}{n-1} \sum_{i=1}^n w_i w_j \sigma_{i,j} \tag{9}$$

Subject to

$$g_1(w_1, w_2, \dots, w_n) = \sum_{i=1}^n w_i \mu_i - \mu_p \geq 0.450\%$$

$$g_2(w_1, w_2, \dots, w_n) = \sum_{i=1}^n w_i - 1 = 0$$

After the simulation, we got a funny result where 100% of our weight is allocated to NEM, with return and SD equal to what we had abinitio and therefore this portfolio is not acceptable.

#### 4.5 Portfolio 4: Maximization of Sharpe ratio

Finally, we maximise the sharpe ratio SR. Here we have the equation as follows

$$\text{maximize } f(w_1, w_2, \dots, w_n) = SR \tag{10}$$

$$g(w_1, w_2, \dots, w_n) = \sum_{i=1}^n w_i - 1 = 0$$

Again, we have the return to be 0.095%, the SD to be 2.06% and SR 4.6%. The weights were loaded in Transcorp, Guaranty trust bank and Custody assets with very few distributed among Skye, Sterling and Wema Banks, others are Wapic and NEM.

Comparison of the results, that is, equally weighted portfolio, Max. Return, Min. Standard deviation and Max SR as shown in Table 3

	Portfolios			
	Equal Wt	Max Return	Min St Dev	Max SR
$\mu_p$	0.00162%	0.084%	0.45%	0.095%
$\sigma_p$	1.28%	1.84%	17.89%	2.06%
$\mu/\sigma$	0.13%	4.56%	2.52%	4.60%

Table 3.A table showing the return, risk and sharpe ratio of the four portfolios constructed.

If we take the equally weighted portfolio as our pivotal portfolio, with return, standard deviation and sharpe ratio as 0.00162%, 1.28% and 0.13% respectively, we notice that it return was below expectations. Though the risk is very minimal but the return and the sharpe ratio show that it is not a good idea to invest in the sector with an equally weighted portfolio. The portfolio that minimizes standard deviation has the highest return but the risk is too much and the sharpe ratio is not encouraging, also the simulation allocated 100% of the weight to one stock (NEM) which does not encourage diversification of funds. Therefore, these make it not healthy for investment. We are now left with two options which are, Max Return and Max SR which have their returns as multiples of 52 and 59 of the return of the equally weighted portfolio respectively. Though the risk value of both is greater than the value of the equally weighted portfolio but the sharpe ratios are better, which again are multiples of 46 on approximate of the equally weighted portfolio.





## 5. Conclusion

We were able to construct four portfolios as we can see the summary in Table 3. The first one is equally weighted and the return is so small that an investor who wants a profit will not be advised to invest in such portfolio.

Secondly, the portfolio that was formed with equation (9) gave the highest return with a very high standard deviation which is not encouraging. Besides, the idea of diversification was killed because the whole fund was allocated to one stock which is NSE. Therefore we advise investors to disregard this. Finally, the equations (8) and (10) gave us something closer to what we want, an appreciable return and a risk that can be tolerated and above all, their sharpe ratios are within acceptable boundaries when compared with the former two. Investors who want to invest in this sector are advised to invest in the portfolio of equation (10), which we consider to be the optimal portfolio. Though, the risk is slightly above the other but the return and the sharpe ratio are very encouraging. Furthermore, we can see from the interaction of the stocks in the correlation matrix (fig 2), that the assets selected in the portfolio move in such direction that will reduce risk. Investors are highly advised to invest in this portfolio.

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