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Approximate Solution of the System of Linear Fractional Integro-Differential Equations of Volterra Using B- Spline Method

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Abstract

The aim of this paper is to introduce new method for solving a system of linear fractional Integro-differential equations of Volterra (LFIDEV) numerically presented based on B-spline techniques. Also, to get reasonable results in this method, convergence and stability have been investigated. Numerical examples of the pertinent features of the method with the proposed system are illustrated.

Keyword: System of fractional derivatives of Volterra equation, fractional Integro-differential equations, B- spline functions.

1. Introduction

The theory and application of fractional integrals and derivatives can been found in many fields of science and engineering such as viscoelasticity, fractional differential operators which have been used to describe materials constitutive equations; see [4, 5, and 10]. There are many methods that concerted of finding the approximated solution of system fractional differential equations; see [2]. This paper divided to the approximate solution for a system of LFIDEV of orders which takes the following general form

$$D_t^{\beta_i} x_i(t) = g_i(t) + \sum_{j=1}^{m} \int_0^t k_{ij}(t,s) x_j(s) ds \quad ,i = 1,2,...,m \quad , \; n-1 < \beta_i < n \quad ,t \in [a,b] \qquad (1)$$

Where $g_i \quad and \quad k_{ij}$ are continuous and bounded functions?

The main aim of this work is to establish a new methods to treat the system (1) using approximate method to a achieve this, we use the properties of B-spline function, here, we consider the fractional given by Riemann-Liouvill definition[10] of α^{th} order fractional derivative with respect to *t* is:

where $n-1 \le \alpha \le n \; ,n \in N$

There are many methods that concerted of finding the approximated solution of system fractional Integro-differential

$$D_a^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} f(s) ds$$

Equations. Hence we prove a new general formula of system fraction order differential equations where most studies have dealt just with special cases; see [1, 3, 4, 8, 9, 11, 12]. The existence and uniqueness of solution of system of LFIDEV have been considered by many authors; see [6, 7, and 13].

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2 Preliminaries

In this section, present some definitions and lemmas to be used later system of LFIDEV

Definition (2.1) (Bazier Curves) [2]

Let P ={P$_0$,P$_1$,…,P$_n$} be a set of point $P_j \in R^2$.The Bazier Curve associated with the set P defined by:

$$P^n(t)=\sum_{j=0}^{n} P_j B_j^n(t) \quad where$$

$$B_j^n(t)=\binom{n}{j}(1-t)^{n-j}t^j \qquad j=0,1,\cdots,n \qquad for \;\; 0\le t\le 1 \qquad (2)$$

Definition (2.2) (n^{tk} degree $B-spline$)

The formula for n^{tk} degree $B-spline$ functions is

$$p^n(t)=\sum_{j=0}^{n} p_j \binom{n}{j}(1-t)^{n-j}t^j \quad , \quad 0\le t\le 1 \qquad (3)$$

Lemma (2.1): Consider the B-Spline curve defined in (3) then the k-th derivative for $n \ge 1$ can be evaluated as:

$$\frac{d^k p^n(t)}{dt^k}=\sum_{l=0}^{n-k}\left[\frac{\prod_{j=0}^{l+k-1}(n-j)}{l!}(1-t)^{n-l-k}t^l\sum_{r=0}^{k}(-1)^{k+r}\binom{k}{r}p_{l+r}\right] for \quad k=1,2,\ldots,n \qquad ,0\le t\le 1 \qquad (4)$$

Proposition (2.1): For $n\ge 1$, the control points P$_k$ can be written as:

$$p_k=\begin{cases} p^n(0) & ; \; if \quad k=0 \\ \sum_{i=1}^{k}\frac{\binom{k}{i}}{\prod_{j=1}^{i-1}(n-j)} \frac{d^i p^n(0)}{dt^i}+p(0) & ; if \;\; k=1,2,3,\ldots,n-1 \\ p^n(1) & ; if \quad k=n \end{cases} \qquad (5)$$

3. The mean results

Proposition (3.1): The general form of n^{th} degree B-spline function can be written as:

$$p^n(t)=\sum_{l=0}^{n-1}\frac{1}{l!}\frac{d^l p^n(0)}{dt^l}(1-t^{n-l})t^l+t^n p^n(1) \qquad ; \; for \;\; n\ge 1 \;\; and \;\; 0\le t\le 1 \qquad (6)$$

Proof: From (3) and (5)

$$p^n(t)=(1-t)^n p^n(0)+\sum_{i=1}^{n-1}\binom{n}{i}(i-t)^{n-i}t^i\left[\sum_{r=1}^{i}\frac{\binom{i}{r}}{\prod_{j=0}^{r-1}(n-j)}\frac{d^r p^n(0)}{dt^r}+p_o\right]+t^n p^n(1)$$

$$= \sum_{i=0}^{n-1}\binom{n}{i}(1-t)^{n-i}t^i p_0 + \sum_{i=0}^{n-1}\binom{n}{i}(1-t)^{n-i}t^i\sum_{r=0}^{i}\frac{\binom{i}{r}}{\prod_{j=0}^{r-1}(n-j)}\frac{d^r p^n(0)}{dt^r}+t^n p^n(1)$$

$$= \sum_{i=0}^{n-1}\binom{n}{i}(1-t)^{n-i}t^i\ p_o + \sum_{r=1}^{n-1}\sum_{i=0}^{n-r-1}\frac{\binom{n}{i+r}\binom{i+r}{r}}{\prod_{j=0}^{r-1}(n-j)}(1-t)^{n-i-r}\ t^{i+r}\ \frac{d^r p^n(0)}{dt^r}+t^n p^n(1)$$

But

$$\frac{\binom{n}{i+r}\binom{i+r}{r}}{\prod_{j=0}^{r-1}(n-j)}=\frac{\binom{n-r}{i}}{r!}$$

$$p^n(t)=\sum_{r=0}^{n-1}\frac{1}{r!}\left[\sum_{i=0}^{n-r-1}\binom{n-r}{i}\ (1-t)^{n-i-r}\ t^{i+r}\right]\frac{d^r p^n(0)}{dt^r}+t^n p^n(1)$$

Let $l = r + i$

$$p^n(t)=\sum_{i=0}^{n-1}\frac{1}{l!}\frac{d^l p^n(0)}{dt^l}(1-t^{n-l})\ \ t^l + t^n p^n(1)$$

Theorem (3.1).Recall the n^{th} degree B-Spline in eq (6) ,then the fractional derivative of β_i order with respect to t is

$$\underset{t}{\overset{\beta_i}{D}} p^n(t)=\frac{t^{-\beta_i}}{\Gamma(-\beta_i+n+1)}\left[\sum_{l=0}^{k-1}\frac{d^l p^n(0)}{dt^l}\left(\frac{\Gamma(-\beta_i+n+1)}{\Gamma(-\beta_i+l+1)}t^l-\frac{n!}{l!}t^n\right)+n!\,p^n(1)t^n\right] \tag{7}$$

Proof: Take $\underset{t}{\overset{\beta_i}{D}}$ of eq(6) with respect to t

$$\underset{t}{\overset{\beta_i}{D}} p^n(t)=\left[\sum_{l=0}^{n-1}\frac{1}{l!}\frac{d^l p^n(0)}{dt^l}\left(\underset{t}{\overset{\beta_i}{D}}(t^l-t^n)\right)+p^n(1)\underset{t}{\overset{\beta_i}{D}}t^n\right]$$, i = 1,2, … , m

$$\underset{t}{\overset{\beta_i}{D}}(t-a)^q=\begin{cases}\dfrac{\Gamma(q+1)(t-a)^{q-\beta_i}}{\Gamma(q-\beta_i+1)} & ;q>-1\ ;\ \beta_i>0\\[2ex] \dfrac{\Gamma(q+1)(t-a)^{q+\beta_i}}{\Gamma(q+\beta_i+1)} & ;q>-1\ ;\ \beta_i>0\end{cases}$$

Since

$$\underset{t}{\overset{\beta_i}{D}} p^n(t)=\sum_{l=0}^{k-1}\frac{1}{l!}\frac{d^l p^n(0)}{dx^l}\left(\frac{l!}{\Gamma(-\beta_i+l+1)}t^{-\beta_i+l}-\frac{n!}{\Gamma(-\beta_i+n+1)}x^{-\beta_i+n}\right)+p^n(1)\frac{n!}{\Gamma(-\beta_i+n+1)}t^{-\beta_i+n}$$

$$= \frac{t^{-\beta_i}}{\Gamma(-\beta_i+n+1)}\left[\sum_{l=0}^{k-1}\frac{d^l p^n(0)}{dt^l}\left(\frac{\Gamma(-\beta_i+n+1)}{\Gamma(-\beta_i+l+1)}t^l - \frac{n!}{l!}t^n\right)+n!\,p^n(1)t^n\right]$$

Theorem (3.2) : If $\underset{t}{\overset{\beta_i}{D}}x(t) = g(t) + \int_0^t k(t,s)x(s)ds \qquad ; n-1<\beta_i<n$, then

$$\underset{t}{\overset{\beta_i}{D}}x^{(r)}(t) = \frac{x^{(r)}(0)t^{-\beta_i}}{\Gamma(-\beta_i+1)} + \frac{1}{\Gamma(-\beta_i+n)}\frac{d^{n-1}}{dt^{n-1}}\int_0^t (t,s)^{n-\beta_i-1}x^{(r+1)}(s)ds \;\; ; r=0,1,2,.... \tag{8}$$

Proof: since the fractional derivative for $x^{(r)}(t)$ *of order* $\beta_i>0$ given by

$$\underset{t}{\overset{\beta_i}{D}}x^{(r)}(t) = \frac{1}{\Gamma(-\beta_i+n)}\frac{d^n}{dt^n}\int_0^t (t,s)^{n-\beta_i-1}x^{(r)}(s)ds$$

$$= \frac{1}{\Gamma(-\beta_i+n+1)}\frac{d^n}{dt^n}x^{(r)}(0)t^{n-\beta_i} + \int_0^t (t,s)^{n-\beta_i}x^{(r+1)}(s)ds$$

$$= \frac{x^{(r)}(0)}{\Gamma(-\beta_i+n+1)}\frac{d^n}{dt^n}t^{n-\beta_i} + \frac{1}{\Gamma(-\beta_i+n+1)}\frac{d^n}{dt^n}\int_0^t (t,s)^{n-\beta_i}x^{(r+1)}(s)ds$$

$$\frac{d^n}{dt^n}t^{n-\beta_i} = \frac{d^{n-1}}{dt^{n-1}}\frac{d}{dt}t^{n-\beta_i} = (n-\beta_i)\frac{d^{n-1}}{dt^{n-1}}t^{n-\beta_i-1} = (n-\beta_i)(n-\beta_i-1)\ldots(-\beta_i+1)t^{-\beta_i}$$

Since

$$= \frac{\Gamma(-\beta_i+n-1)}{\Gamma(-\beta_i+1)}t^{-\beta_i}$$

by fundamental theorem of integral calculus ,yields

$$\underset{t}{\overset{\beta_i}{D}}x^{(r)}(t) = \frac{x^{(r)}(0)t^{-\beta_i}}{\Gamma(-\beta_i+1)} + \frac{1}{\Gamma(-\beta_i+n)}\frac{d^{n-1}}{dt^{n-1}}\int_0^t (t,s)^{n-\beta_i-1}x^{(r+1)}(s)ds \; .$$

Theorem(3.3) : If $\underset{t}{\overset{\beta_i}{D}}x(t) = g(t) + \int_0^t k(t,s)x(s)ds \qquad ; n-1<\beta_i<n$, (9)

then

$$x^{(r)}(0) = \left(t^{\beta_i}\Gamma(-\beta_i+1)g^{(r)}(t) - \Gamma(-\beta_i+1)\sum_{k=1}^{r}\frac{t^{-k}x^{(r-k)}(0)}{\Gamma(-\beta_i-k+1)}\right)_{t=0} \quad ; for\,all\;\; r=0,1,2,.\cdots \tag{10}$$

Proof: Differential both sides of (9) r times with respect to t , to get:

$$\frac{d^r}{dt^r}\underset{t}{\overset{\beta_i}{D}}x(t) = g^{(r)}(t) + \frac{d^r}{dt^r}\int_0^t k(t,s)x(s)ds$$

, then using (8) , yields

$$\frac{x^{(r)}(0)}{\Gamma(-\beta_i+1)}t^{\beta_i} + \frac{1}{\Gamma(-\beta_i+n)}\frac{d^{n-1}}{dt^{n-1}}\int_0^t (t-s)^{n-\beta_i-1}x^{(r+1)}(s)ds + \sum_{k=1}^{r}\frac{t^{-\beta_i-k}x^{(r-k)}(0)}{\Gamma(-\beta_i-k+1)}$$

$$= g^{(r)}(t) + \frac{d^r}{dt^r}\int_0^t k(t,s)x(s)ds$$

Finally, by multiplying by $t^{\beta_i}\Gamma(-\beta_i+1)$ and put t = 0, the proof complete .

Lemma (3.1) (Stability of B- Spline functions)

The form of n^{th} degree B- Spline functions is stable if *cond* (A) =1.

Proof: From (4) and (5)

$$p^n(0) = p_0\,;\quad \frac{1}{\prod_{j=0}^{r-1}(n-j)}\frac{d^r p^n(0)}{dt^r} = \sum_{l=0}^{r}(-1)^{l+r}\binom{r}{l}\; p_l\;;r=1,2,\ldots,n-1\,;n\ge 1 \, and\, p^n(1)=p_n \quad . \qquad (11)$$

The n^{th} degree B- spline functions, can be written by the form AP = B where

$$A = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ -1 & 1 & 0 & \ldots & 0 & 0 \\ 1 & -2 & 1 & \ldots & 0 & 0 \\ -1 & 3 & -3 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (-1)^{n-1} & (-1)^n\binom{n-1}{2} & (-1)^{n+1}\binom{n-1}{2} & \ldots & 1 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 1 \end{pmatrix} ;P = \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_n \end{bmatrix} ;B = \begin{bmatrix} p^n(0) \\ \frac{1}{n}\frac{d}{dt}p^n(0) \\ \frac{1}{n(n-1)}\frac{d^2}{dt^2}p^n(0) \\ \vdots \\ \frac{1}{\prod_{j=0}^{n-2}(n-j)}\frac{d^{n-1}}{dt^{n-1}}p^n(0) \\ p^n(1) \end{bmatrix}$$

the matrix A is (n+1)×(n+1) lower triangular , so we need only to show that $|a_{ii}| \le 1 \quad for\,all\;\; i=1,2,3,\ldots,n+1$, where a_{ii} the diagonal element of A in raw i. Since a_{ii} =1 for all i, hence the scheme is unconditionally stable , this mean $cond(A)=1$.

4. Numerical method

In this sections, B- spline function of degree n, n = 1,2,3,…. Will be used to find the approximation solution of Eq. (1) .

Approximate $x_i(t)\;\;;\; i=1,2,\ldots,m$ using (6)

$$x_i(t) = p_i^n(t) = p_i^{\,n}(0)(1-t^n) + \frac{d}{dt}p_i^{\,n}(0)(t-t^n) + \frac{1}{2!}\frac{d^2}{dt^2}p_i^{\,n}(0)(t^2-t^n) + \ldots$$

$$+ \frac{1}{(n-1)!}\frac{d^{n-1}}{dt^{n-1}}p_i^{\,n}(0)(t^{n-1}-t^n) + p_i^{\,n}(1)t^n \quad ;0\le t\le 1$$

Substitution in (1) ,yields

$$\overset{\beta_i}{\underset{t}{D}}\left[p_i^{\,n}(0)(1-t^n)+\frac{d}{dt}p_i^{\,n}(0)(t-t^n)+\frac{1}{2!}\frac{d^2}{dt^2}p_i^{\,n}(0)(t^2-t^n)+\ldots+\frac{1}{(n-1)!}\frac{d^{n-1}}{dt^{n-1}}p_i^{\,n}(0)(t^{n-1}-t^n)+p_i^{\,n}(1)t^n \right]$$

$$=g_i(t)+\sum_{j=1}^{m}\int_0^t k_{ij}(t,s)[p_j^{\,n}(0)(1-s^n)+\frac{d}{dt}p_j^{\,n}(0)(s-s^n)+\frac{1}{2!}\frac{d^2}{dt^2}p_j^{\,n}(0)(s^2-s^n)+\ldots$$

$$+\frac{1}{(n-1)!}\frac{d^{n-1}}{dt^{n-1}}p_j^{\,n}(0)(s^{n-1}-s^n)+p_j^{\,n}(1)s^n]ds \qquad (12)$$

Use eqs (6, 7, 9, and 10) the optained resalt is used to determine the unknown parameters:

$p_i^{\,n}(0)$, $\frac{d}{dt}p_i^{\,n}(0)$, $\frac{d^2}{dt^2}p_i^{\,n}(0)$,… , $\frac{d^{n-1}}{dt^{n-1}}p_i^{\,n}(0)$ *and* $p_i^{\,n}(1)$, $i=1,2,\ldots,m$ as following:

$$\frac{d^r}{dt^r}p_i^{\,n}(0)=\left[t^{-\beta_i}\Gamma(-\beta_i+1)g^{(r)}(t)-\Gamma(-\beta_i+1)\sum_{j=1}^{r}\frac{t^{-j}}{\Gamma(-\beta_i-j+1)}\frac{d^{r-1}}{dt^{r-1}}p_i^{\,n}(0)\right]_{t=0} \quad ;r=0,1,\ldots,n-1 \qquad (13)$$

eq (12) can be written by $Ap^n=B$ *where* $A=(a_{ij})$; $i,j=1,2,\ldots,m$ (14)
and

$$a_{ij}=\begin{cases} -\frac{1}{n!}\Gamma(-\beta_i+n+1)\int_0^1 k_{ij}(1,s)s^n ds & ,i\neq j \\ 1-\frac{1}{n!}\Gamma(-\beta_i+n+1)\int_0^1 k_{ij}(1,s)s^n ds & ,i=j \end{cases} \qquad (15)$$

$p^n=[p_1^{\,n}(1),p_2^{\,n}(1),p_3^{\,n}(1),\ldots,p_m^{\,n}(1)]$, and B = (b_i) ,where

$$b_i=\frac{1}{n!}\sum_{r=0}^{n-1}\frac{d^r}{dt^r}p_i^{\,n}(0)\left(\frac{n!}{r!}-\frac{\Gamma(-\beta_i+n+1)}{\Gamma(-\beta_i+r+1)}\right)+\frac{1}{n!}\Gamma(-\beta_i+n+1)\left[g_i(1)+\sum_{j=1}^{m}\left(\int_0^1 k_{ij}(1,s)\sum_{r=0}^{n-1}\frac{1}{r!}\frac{d^r}{dt^r}p_j^n(0)(s^r-s^n)ds\right)\right]$$

Compute $\int_0^1 k_{ij}(1,s)\sum_{r=0}^{n-1}\frac{1}{r!}\frac{d^r}{dt^r}p_j^n(0)(s^r-s^n)ds$ *and* $\int_0^1 k_{ij}(1,s)ds$ by (Newton-Cotes method)
Then Eq. (14) can be solved by Gauss elimination method.

5 The convergence of this method

Theorem (5.1): Assume that the existence and uniqueness solution for (1) is exist. If $t_r=t_0+rh$, $r=0,1,2,\ldots,l$ *and* $0\le h\le T_{\min}$; $|k_{ij}(x,t)|<N$; $i,j=1,2,\cdots,m$ (16)

where $T_{\min}=\left\{\frac{T}{l},1\right\}$ *and* $\left(\frac{(m-2)\Gamma(-\beta_i+n+1)}{(n+1)!}N\right)^{n+1}$ (17)

then, the B-spline function method converges to the unique solution of equation (1)

Proof

Since $0 \le t \le 1 \quad and \quad t_r = rh \quad , r = 0,1,2,\ldots,l$,substitute in eq(15) yields

$$a_{ij} = \begin{cases} -\dfrac{1}{n!}\Gamma(-\beta_i + n + 1)\displaystyle\int_0^{lh} k_{ij}(lh,s)s^n ds & ,i \ne j \\ 1 - \dfrac{1}{n!}\Gamma(-\beta_i + n + 1)\displaystyle\int_0^{lh} k_{ij}(lh,s)s^n ds & ,i = j \end{cases}$$

For i=1,2,…,m

$$\sum_{\substack{j=1\\ j\ne i}}^{m}\left|a_{ij}\right| = \sum_{\substack{j=1\\ j\ne i}}^{m}\left|\frac{1}{n!}\Gamma(-\beta_i + n+1)\int_0^{lh} k_{ij}(lh,s)s^n ds\right| \le \sum_{\substack{j=1\\ j\ne i}}^{m}\left|\frac{\Gamma(-\beta_i + n+1)N(lh)^{n+1}}{(n+1)!}\right| \le (m-1)Q(h) \qquad (18)$$

and $|a_{ii}| \ge |1 - Q(h)|$ (19)

where $Q(h) = \dfrac{\Gamma(-\beta_i + n+1)N(lh)^{n+1}}{(n+1)!}$

$$\sum_{\substack{j=1\\ j\ne i}}^{m}\left|a_{ij}\right| < \left|a_{ii}\right| \qquad for \quad i = 1,2,\ldots,m \qquad (20)$$

From (16) and(17) yields $(m-1)Q(h) < |1 - Q(h)|$ (21)

And the condition (20) is satisfied.

6. Numerical examples

In this section, Example of numerical solution of Eq (1) by means of the approximate described in sections (3 - 4) is given. To better analyze the numerical results, we have chosen those problem which have analytical solutions of simple type.

Example (1): Consider the system of *LFIDEV's*

$$\underset{t}{\overset{0.25}{D}} x_1(t) = \frac{1}{\Gamma(1.75)} t^{\frac{3}{4}} - e^t - \frac{t^2}{2} - \frac{t^4}{3} + 1 + \int_0^t [(1-ts)(x_1(s) + x_2(s))]ds$$

$$\underset{t}{\overset{0.5}{D}} x_2(t) = \frac{2}{\sqrt{\pi}} t^{-\frac{1}{2}} + e^t erf(\sqrt{t}) - e^t - te^t - t - \frac{t^2}{2} + 1 + \int_0^t [(1+t)x_2(s) + x_3(s)]ds$$

$$\underset{t}{\overset{0.75}{D}} x_3(t) = \frac{2}{\Gamma(0.25)} t^{-\frac{3}{4}} + \frac{1}{\Gamma(1.25)} t^{\frac{1}{4}} - \frac{t^2}{2} + t - 4e^t - te^t + 4 + \int_0^t [x_1(s) + 2x_2(s) + e^{t-s}x_3(s)]ds$$

where the analytical solution $x_1(t) = t \quad ; \quad x_2(t) = e^t \quad and \quad x_3(t) = 2 + t$

Table 1: the error decreases substantially of the B- spline degree is increased for example (1)

B- spline	p^1	p^2	p^3	p^4
L.S.E $x(t)$	0.76835	0.12953	0.04283	$1.1201e^{-004}$

Table 2: the numerical results obtained for example (1) when h = 0.1

t	$\left\lvert x_1(t)-p_1^4(t)\right\rvert$	$\left\lvert x_2(t)-p_2^4(t)\right\rvert$	$\left\lvert x_3(t)-p_3^4(t)\right\rvert$
0	0	0	0
0.1	0.000329	0.0028	0.000333
0.2	0.000648	0.0046	0.000661
0.3	0.000946	0.0053	0.000973
0.4	0.0012	0.0051	0.0012
0.5	0.0014	0.0042	0.0015
0.6	0.0015	0.0028	0.0016
0.7	0.0015	0.0015	0.0015
0.8	0.0013	0.0013	0.0013
0.9	0.000802	0.0000778	0.0008164
1.0	0.000064	0.000482	0.0000052
L.S.E	$1.1456e^{-005}$	$1.1201e^{-004}$	$1.236e^{-005}$

Table 3: Comparison between the exact solution and the numerical solution of Example (1) with different value of *h*

L.S.E	h = 0.1	h = 0.05	h = 0.01
$\left\lVert x_1(t)-p_1^4(t)\right\rVert$	$1.1456e^{-005}$	$4.1354e^{-007}$	$1.7283e^{-007}$
$\left\lVert x_2(t)-p_2^4(t)\right\rVert$	$1.1201e^{-004}$	$7.3138e^{-005}$	$2.5294e^{-006}$
$\left\lVert x_3(t)-p_3^4(t)\right\rVert$	$1.236e^{-005}$	$2.7203e^{-006}$	$3.1157e^{-007}$

Conclusions

According to the numerical results which obtaining from the illustrative example, we conclude that for sufficiently small *h* we get a good accuracy, since by reducing step size length the least square error will be reduced.

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