Restructuring Debt Model with Equivalent Equations: Theoretical and Practical Implications

Arturo García-Santillán¹ Francisco Venegas-Martínez² & Milka Elena Escalera-Chávez³

Abstract

This paper is aimed at developing a restructuring debt model between debtor and creditor. To do this, we use several hypothetic scenarios with some expired promissory notes and other documents which are not yet expired. The proposed equivalent equation model is useful to examine the three moments of the restructuring: valuing of the original debt, determining the new payment scheme, and computing new payments.

Keywords and Phrases: Restructuring debt, Equivalent equation, Promissory notes

AMS Subject Classification: 62P05, 97M30

1. Introduction

The debt restructuring between debtors and creditors (clients, suppliers, companies) may sometimes be not so fair for one of the parties. That is, when there is no enough money to pay a previously acquired debt with some supplier of goods and/or services, it may be present an insolvency situation of the debtor for the payment of his/her obligation ([1, 3, 4]). This is very common when the cash flows have not been correctly budgeted, which could result in a payment default. In most cases, each promissory note which is due (owed), bring with it a small lapse of extension of time for delay.

¹ProfessorResearcher at Universidad Cristóbal Colón, Veracruz México and Postdoctoral ResearchFellow at Escuela Superior de Economía-IPN México. Email: arturogarciasantillan@yahoo.com.mx
²Escuela Superior de Economía, Instituto Politécnico Nacional (ESE-IPN) México. Email: fvenegas1111@yahoo.com.mx
³ProfessorResearcher at Universidad Autónoma de San Luis Potosí, SLP México, Posdoctoral ResearchFellow at ESE-IPN México. Email: milkaech@uaslp.mx
We are concerned, in this research, with developing a mathematical model that makes possible establishing a restructuring debt scheme between debtor and creditor resulting both benefitted. In this regard, García-Santillán and Vega-Lebrúm [4] suggested a scheme to establish a debt restructuring by a new payment schedule that best fits the needs of the debtor, provided that the creditor agrees. Both parts (debtor and creditor) should arrive at a consensus, in which everyone should be benefited from this agreement.

Regarding this, we answer the following question: Is there a model that allows calculate a debt restructuring in a way equitably for both parties? The variables of this study are displayed in Figure 1.

Figure 1: Component of Restructuring Debt (Adapted from García-Santillán and Vega-Lebrúm, 2008)

Right after obtaining any kind of credit, the credit immediately becomes a debt to whom receives the resource, beginning with this fact the relationship between the debtor and the creditor. We start the modeling with equivalent equations under the assumption that the debtor does not have to pay o the credit granted by the creditor [4, 5, 7]. In this regard, for instance, Citibank [8] raises a number of considerations which are associated with restructuring debt. First, we suppose that there are not problems of nonperforming loans. There are, of course, other factors which may be associated with the problem of debt default, for instance, financial crises.

Budgeting is a recommended practice because it is a tool which helps to organize and control flows, as indicated by Citibank [8].
It is clear that budgeting will not help to get out of debt; however, it will help to identify those expenses which are not necessary to get a reserve of money to amortize debt. It is always more advisable to seek a renegotiation or restructuring before a moratorium, suspension of payments, or bankruptcy declaration [4]. Citibank refers to this textually:

**Definition 1. Negotiate with your creditor**: "Do not fear to negotiate with your creditor to devise a way to pay your debt: Design a plan to be presented to your creditors. It is recommended to pay the same amount each period to each creditor, or paying a little more to the creditor that charges the higher interest rate" [8].

From the above considerations, we develop a model that allow us to compute an equitable debt restructuring by using equivalent equations.

Scenarios to model:

a) Equivalent equations

Next, we will calculate a new payment scheme with equal payments through equivalent equations. From now on, we will denote the payment for each period by X. For the sake of simplicity, X takes the constant value of 1[2, 3, 6]. Once we get the value of VOD (valuation of original debt) and VNS (valuation of new scheme), we use them to get value of each equal payment (Y ) provided the payments have been agreed. If it is not the case, the new payments are calculated based on the agreed values, and the approach is different.

1.1 A formula for Restructuring Debt with Equivalent Equations

As usual, the formula that allows us to value the original debt with compounded interest rate is as follows:

\[
V_{OD} = \sum_{1}^{D_{b/f_d}} \left(1 + \frac{i_{n\cdot d}}{m}\right)^{\frac{t}{m}} + D_{b/f_d} \left(1 + \frac{i_{n\cdot d}}{m}\right)^{\frac{t}{m}} \ldots \\
\ldots \frac{D_{b/f_d \cdot n}}{1 + \frac{i_{n\cdot d}}{m}} \left(1 + \frac{i_{n\cdot d}}{m}\right)^{\frac{t}{m}} + D_{f_d} + \sum_{1}^{D_{a/f_d}} \frac{D_{a/f_d \cdot n}}{1 + \frac{i_{d}}{m}} \left(1 + \frac{i_{d}}{m}\right)^{\frac{t}{m}} + \frac{D_{a/f_d \cdot n}}{1 + \frac{i_{d}}{m}} \left(1 + \frac{i_{d}}{m}\right)^{\frac{t}{m}} \ldots \\
\ldots \frac{D_{a/f_d \cdot n}}{1 + \frac{i_{d}}{m}} \left(1 + \frac{i_{d}}{m}\right)^{\frac{t}{m}}
\]
After this, we calculate a new scheme with the next formula:

\[
V_{NS} = \sum_{1 \leq n}^{D_{bfd1-n}} X_{bfd1}(1 + \frac{i_{indx}}{m})^{\Sigma t/m} + X_{bfd2}(1 + \frac{i_{indx}}{m})^{\Sigma t/m} \ldots
\]

\[
\ldots X_{bfd_n}(1 + \frac{i_{indx}}{m})^{\Sigma t/m} + X_{fd} + \sum_{1 \leq n}^{D_{bfd1-n}} \frac{X_{afd1}}{(1 + \frac{i_d}{m})^{\Sigma t/m}} + \frac{X_{afd2}}{(1 + \frac{i_d}{m})^{\Sigma t/m}} \ldots
\]

\[
\ldots \frac{X_{afd_n}}{(1 + \frac{i_d}{m})^{\Sigma t/m}}
\]

In all cases \(X\) corresponds to each one of the payments. The resulting formula is:

\[
V_{NS} = \sum_{1 \leq n}^{D_{bfld1-n}} (1 + \frac{i_{indx}}{m})^{\Sigma t/m} + 1_{bfd1}(1 + \frac{i_{indx}}{m})^{\Sigma t/m} \ldots
\]

\[
\ldots 1_{bfd_n}(1 + \frac{i_{indx}}{m})^{\Sigma t/m} + 1_{fd} + \sum_{1 \leq n}^{D_{bfld1-n}} \frac{1_{afd1}}{(1 + \frac{i_d}{m})^{\Sigma t/m}} + \frac{1_{afd2}}{(1 + \frac{i_d}{m})^{\Sigma t/m}} \ldots
\]

\[
\ldots \frac{1_{afld_n}}{(1 + \frac{i_d}{m})^{\Sigma t/m}}
\]

Finally, in order to obtain the each equal payment value, we use

\[
Y = \frac{V_{OD}}{V_{NSP}}
\]
Where:

- \( \text{debr} = \) debtor
- \( \text{crtr} = \) creditor
- \( n = \) time \( \left( \sum t/m \right) \)
- \( m = \) capitalization
- \( I_p = \) Interest borne by the debtor
- \( I_g = \) Interest earned favor of the creditor
- \( i_{dm} = \) Accurate Interest rate (to discount) \( \left( \sum id / 365^*m \right) \)
- \( i_{indx} = \) Accurate Interest rate (to indexing) \( \left( \sum indx / 365^*m \right) \)

- \( \text{fd} = \) focal date
- \( \text{afd} = \) after focal date
- \( \text{bfd} = \) before focal date
- \( D_1, ..., D_n = \) Document\( _1, ..., \) Document\( _n \)
- \( \text{VOD} = \) Valuation original debt
- \( \text{VNSP} = \) Valuation new scheme of payments
- \( Y = \) equal payment

In order to have an illustrative approach to the model, we provide Figure 2, in which the line of time shows the moment when the debt restructuring is established.

**Figure 1: Time Line to Debt Restructuring (Own)**

If we are unable to pay some promissory notes, considering that some of these have already been expired (they have not paid), then the negotiation begins. The focal date is set, the original debt is valued, i.e., the expired promissory notes are valued (indexed), and the promissory notes not expired are valued too. The latter are brought to present value to the focal date by using a discount factor. Subsequently, the new payment schedule is established.
It is therefore necessary to calculate the coefficient of payments which is required for the denominator of the formula \( Y = \frac{V_{DO}}{V_{NE}} \).

Hence, it is important to be aware that the factors for accumulation and discount, and it will be required to have the nominal rate that will become effective for the valuation of the expired promissory notes (the highest rate) and the real interest rate (the lowest rate) for the discounting of not expired promissory notes.

2. Methodology

2.1 Debt Restructuring with Equivalent Equations

In order to compute variable \( Y \) (amount of each payment) in a debt restructuring between debtor and creditor, we use hypothetic scenarios with some expired promissory notes and other promissory notes that are not yet expired. Therefore, we will be using an equivalent equation model to calculate the three moments of the restructuring: valuation of the original debt (\( V_{OD} \)), valuation of the new payment scheme (\( V_{NSP} \)) and calculate the amount of each new payments (\( Y \)).

Moreover, for indexation of all promissory notes which have been expired, we will use the effective interest rate; otherwise, to discount the value for all promissory notes not expired, we utilized the real interest rate.

where:

- \( T_N = \) Nominal interest rate for indexing 12 % (should become to effective interest rate)
- \( T_R = \) Nominal interest rate for discounting = unknown (the real interest rate)
- \( T_I = \) Inflation rate 3.5%
- \( m = \) Capitalization every 27 days in both cases: \( V_{OD} \) and \( V_{NSP} \) (indexing and discounting)
2.1.1. Time line path in Restructuring

![Diagram showing time line path in restructuring]

2.1.2. Data for Development an Equivalent Equation Model

<table>
<thead>
<tr>
<th>Pemissory notes</th>
<th>Due date (expired or maturity)</th>
<th>id/ = Accurate Real Interest rate (to discount)</th>
<th>im/ = Accurate Effective Interest rate (to indexing)</th>
<th>Amount (Thousands of dls.)</th>
<th>DATA: Nominal exchange rate 12%, Inflation rate 3.45%, capitalization every 27 days (for all cases), effective interest rate and real interest rate are unknown?</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Expired 171 days ago (bfd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$100.00$</td>
<td>In order to calculate the three moment in debt restructuring ($V_{OD}$, $V_{NS}$ and $Y$), we determine:</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>Expired 163 days ago (bfd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$120.00$</td>
<td>1.- For $V_{OD}$ (valuation original debt), a.- utilize effective interest rate for all document expired (bfd)</td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td>Expired 78 days ago (bfd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$115.00$</td>
<td>b.- utilize real interest rate for all document not expired (afd)</td>
<td></td>
</tr>
<tr>
<td>F4</td>
<td>Expired 35 days ago (bfd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$90.00$</td>
<td>$TR = \frac{(Te - TI)}{(1 + TI)} \ast 100$</td>
<td></td>
</tr>
<tr>
<td>F5</td>
<td>Expired in focal date (fd)</td>
<td>Without interest rate</td>
<td>$71.50$</td>
<td>2.- For $V_{NS}$ (valuation new scheme of payment), a.- utilize real interest rate for all new scheme of payment before focal date and after focal date</td>
<td></td>
</tr>
<tr>
<td>F6</td>
<td>Debt with a maturity of 31 days (afd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$111.00$</td>
<td>$TR = \frac{(Te - TI)}{(1 + TI)} \ast 100$</td>
<td></td>
</tr>
<tr>
<td>F7</td>
<td>Debt with a maturity of 67 days (afd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$123.00$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F8</td>
<td>Debt with a maturity of 81 days (afd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$200.00$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F9</td>
<td>Debt with a maturity of 131 days (afd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$300.00$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F10</td>
<td>Debt with a maturity of 171 days (afd)</td>
<td>$(\sum i_{id} / 365^* m)$</td>
<td>$190.00$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: (provided by the authors)
2.1.3. The New Scheme of Equal Payments is as Follow

<table>
<thead>
<tr>
<th>Promissory notes</th>
<th>Date (before, in focal date or after)</th>
<th>Equal payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pn</td>
<td>25.65 days before focal date</td>
<td></td>
</tr>
<tr>
<td>Pn1</td>
<td>19.5 days before focal date</td>
<td></td>
</tr>
<tr>
<td>Pn2</td>
<td>7.87 days before focal date</td>
<td></td>
</tr>
<tr>
<td>Pn3</td>
<td>In the focal date</td>
<td></td>
</tr>
<tr>
<td>Pn4</td>
<td>1.5 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn5</td>
<td>45 days after focal date</td>
<td>20 equal payments</td>
</tr>
<tr>
<td>Pn6</td>
<td>75 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn7</td>
<td>115 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn8</td>
<td>120 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn9</td>
<td>150 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn10</td>
<td>168 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn11</td>
<td>181 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn12</td>
<td>197 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn13</td>
<td>245 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn14</td>
<td>270 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn15</td>
<td>297 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn16</td>
<td>320 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn17</td>
<td>348.75 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn18</td>
<td>370.189 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn19</td>
<td>500 days after focal date</td>
<td></td>
</tr>
<tr>
<td>Pn20</td>
<td>500 days after focal date</td>
<td></td>
</tr>
</tbody>
</table>

Source: (provided by the authors)

3. Theoretical and Empirical Results

Now, we proceed to highlight the findings in the proposed model of debt restructuring. First, we calculated the indexation with the discount rates to bring each one of the promissory notes (due and overdue) to the focal date. Starting from the nominal rate, the effective rate for indexing the promissory notes due is calculated and subsequently the real interest rate will be used to discount the documents that are not yet due. These interest rates are used to value the original debt ($V_{OD}$) and to value a new scheme of payment ($V_{NSP}$) to finally calculate the amount of each new payment ($Y$) from the following formula:
3.1 Debt Restructuring

First, we calculate indexing and discounting factors for VOD (valuation of original debt)

1.- For indexing the effective interest rate is used for all expired promissory notes (bfd)

\[ Te = \left( \left( 1 + \frac{i}{m} \right)^{n/m} - 1 \right) \times 100 \]

\[ = \left( \left( 1 + \left( \frac{12 \times 27}{365} \right)^{365/27} - 1 \right) \times 100 \right) \]

\[ = \left( \left( 1 + (0.00887671)^{13.5185185} - 1 \right) \times 100 = [1.126899997 - 1] \times 100 \right) \]

\[ Te = 12.6899997 \]

2.- For discounting the real interest rate is used for all not expired promissory notes (afd)

\[ TR = \frac{(Te - TI)}{(1 + TI)} \times 100 = \frac{(1.26899997 - 0.035)}{1.035} = \frac{(0.009189997)}{1.035} \]

\[ = 0.08879227 \times 100 \]

\[ TR = 8.879227 \]

Now, to calculate the valuation of new payment scheme (V_{NSP}), we use the same factors as in V_{OD}

1.- For indexing the effective interest rate is used for all new promissory notes that shall pay before focal date (bfd)

\[ Te = \left( \left( 1 + \frac{i}{m} \right)^{n/m} - 1 \right) \times 100 \]
\[ Te = 12.6899997 \]

2. For discounting the real interest rate is used for all new promissory notes that shall pay after focal date (afd)

\[ TR = \frac{(Te - TI)}{(1 + TI)} \times 100 \]

\[ TR = 8.879227 \]

Finally, we calculate the interest rate capitalization every 27 days utilizing \( Te \) and \( TR \) as follow:

For \( V_{NSP} \) all promissory notes that will pay before focal date

\[ i_{\text{capitalization}} = \left( \frac{0.12689997 \times 27}{365} \right) = 0.00938712 \]

For \( V_{NSP} \) all promissory notes that will pay after focal date

\[ i_{\text{capitalization}} = \left( \frac{0.08879227 \times 27}{365} \right) = 0.0065682 \]

3.2 Valuation of Original Debt

To value the original debt of all promissory notes (expired) in the focal date, we use the indexation factor, and to value the original debt of all promissory notes (not expired) we bring them from the future to the focal date all promissory notes. To do this, we use the discount factor. Hence, it is necessary compute the interest rate (capitalization every 27 days) from the effective interest rate and real interest rate, respectively. Thus, it follows that
\[ V_{OD} = \sum_{i=1}^{n} D_{bdf_i} \left(1 + \frac{i_{ind}}{m}\right)^{\Sigma t/m} + D_{bfd_i} \left(1 + \frac{i_{ind}}{m}\right)^{\Sigma t/m} \ldots \]

\[ \ldots + D_{bfd_n} \left(1 + \frac{i_{ind}}{m}\right)^{\Sigma t/m} + D_{fd} + \sum_{i=1}^{n} \frac{D_{afd_i}}{(1 + \frac{i_d}{m})^{\Sigma t/m}} + \frac{D_{afd_n}}{(1 + \frac{i_d}{m})^{\Sigma t/m}} \ldots \]

\[ \frac{D_{afd_n}}{(1 + \frac{i_d}{m})^{\Sigma t/m}} \]

\[ V_{OD} = \sum_{i=1}^{n} \left(100.00(1 + 0.00938712)^{17/27} + 120.00(1 + 0.00938712)_{163/27} + \ldots \right. \]

\[ \ldots + 115.00(1 + 0.00938712)_{78/27} + 90.00(1 + 0.00938712)_{35/27} + 71.50 + \ldots \]

\[ \sum_{i=1}^{n} \left(100.00(1 + 0.0065682)^{131/27} + 123.00(1 + 0.0065682)^{67/27} + 200.00(1 + 0.0065682)^{81/27} \ldots \right. \]

\[ \ldots + 111.00(1 + 0.0065682)_{131/27} + 190.00 \right) \]

\[ V_{OD} = \sum_{i=1}^{n} \left(100.00(1.00938712)^{6.333333} + 120.00(1.00938712)^{06.0703704} + \ldots \right. \]

\[ \ldots + 115.00(1.00938712)^{2.888889} + 90.00(1.00938712)^{1.29629630} + 71.50 + \ldots \]

\[ \sum_{i=1}^{n} \left(100.00(1.0065682)^{1.14814815} + 123.00(1.0065682)^{2.48148148} + 200.00(1.0065682)^{3} + \ldots \right. \]

\[ \ldots + 111.00(1.0065682)^{133/27} + 190.00 \right) \]

\[ V_{OD} = \sum_{i=1}^{n} \left(100.00(1.06096031) + 120.00(1.05802722) + \ldots \right. \]

\[ \ldots + 115.00(1.02735944) + 90.00(1.01218537) + 71.50 + \ldots \]

\[ \sum_{i=1}^{n} \left(100.00(1.00754493) + 123.00(1.01637825) + 200.00(1.01983431) + \ldots \right. \]

\[ \ldots + 111.00(1.03227358) + 190.00 \right) \]
\[ V_{OD} = \sum_{D_{m,n}} \sum_{1,n}^{D_{m,n}} $106.10 + $126.96 + $118.15 + $91.1 + $71.50 + ... \\
... + \sum_{1,n}^{D_{m,n}} $110.17 + 121.02 + 196.11 + 290.62 + 182.28 \\
\]

\[ V_{OD} = $1,414.00 \]

In this way, we obtain the total value of original debt.

### 3.3 Valuation of the New Scheme of Payments

To assess the new scheme of payment, we use the indexation factor for all new payment before the focal date and to value all the new payment after the focal date we use the discount factor. It is also necessary utilize the interest rate with capitalization every 27 days, from the effective interest rate and real interest rate, respectively. After this, we assess a new scheme with the following formula:

\[ V_{NSP} = \sum_{b_{m,d}}^{D_{b_{m,d}}} X_{bfd_{m,d}} \left(1 + \frac{i_{indx}}{m}\right) \sum_{j=1}^{t/m} + X_{bfd_{m,d}} \left(1 + \frac{i_{indx}}{m}\right) \sum_{j=1}^{t/m} + ... \\
... + X_{bfd_{m,d}} \left(1 + \frac{i_{indx}}{m}\right) \sum_{j=1}^{t/m} + X_{fd_{m,d}} + \sum_{1,n}^{D_{m,n}} \frac{X_{afd_{m,d}}}{(1 + \frac{i_{d}}{m}) \sum_{j=1}^{t/m}} + ... \\
... + \frac{X_{afd_{m,d}}}{(1 + \frac{i_{d}}{m}) \sum_{j=1}^{t/m}} + \frac{X_{afd_{m,d}}}{(1 + \frac{i_{d}}{m}) \sum_{j=1}^{t/m}} \\
\]

In all cases \( X \) = corresponding to each one of the payment

The above formula becomes:
\[ V_{NSP} = \sum_{i=1}^{D_{bfd}} l_{bfd_i} \left( 1 + \frac{indx}{m} \right)^{\hat{a}t/m} + l_{bfd_1} \left( 1 + \frac{indx}{m} \right)^{\hat{a}t/m} \]

\[ ... l_{bfd_n} \left( 1 + \frac{indx}{m} \right)^{\hat{a}t/m} + l_{fd} + \sum_{i=1}^{D_{afd}} l_{afd_i} \left( 1 + \frac{i d}{m} \right)^{\hat{a}t/m} + ... \]

\[ ... + \frac{l_{afd_2}}{(1 + \frac{i d}{m})^{\hat{a}t/m}} + ... + \frac{l_{afd_n}}{(1 + \frac{i d}{m})^{\hat{a}t/m}} \]

Thus, we have:

\[ V_{NSP} = \sum_{i=1}^{D_{bfd}} l_{bfd_i} \left( 1 + 0.00938712 \right)^{25.65/27} + l_{bfd_2} \left( 1 + 0.00938712 \right)^{19.5/27} + ... \]

\[ ... + l_{bfd_3} \left( 1 + 0.00938712 \right)^{7.87/27} + l_{fd_4} + \sum_{i=1}^{D_{afd}} l_{afd_i} \left( 1 + 0.0065682 \right)^{1.5/27} + ... \]

\[ ... + \frac{l_{afd_6}}{(1 + 0.0065682)^{35/27}} + \frac{l_{afd_7}}{(1 + 0.0065682)^{75/27}} + \frac{l_{afd_8}}{(1 + 0.0065682)^{115/27}} + ... \]

\[ ... + \frac{l_{afd_9}}{(1 + 0.0065682)^{120/27}} + \frac{l_{afd_{10}}}{(1 + 0.0065682)^{150/27}} + \frac{l_{afd_{11}}}{(1 + 0.0065682)^{168/27}} + ... \]

\[ ... + \frac{l_{afd_{12}}}{(1 + 0.0065682)^{181/27}} + \frac{l_{afd_{13}}}{(1 + 0.0065682)^{197/27}} + \frac{l_{afd_{14}}}{(1 + 0.0065682)^{245/27}} + ... \]

\[ ... + \frac{l_{afd_{15}}}{(1 + 0.0065682)^{370/27}} + \frac{l_{afd_{16}}}{(1 + 0.0065682)^{297/27}} + \frac{l_{afd_{17}}}{(1 + 0.0065682)^{320/27}} + ... \]

\[ ... + \frac{l_{afd_{18}}}{(1 + 0.0065682)^{348.75/27}} + \frac{l_{afd_{19}}}{(1 + 0.0065682)^{370.189/27}} + \frac{l_{afd_{20}}}{(1 + 0.0065682)^{500/27}} \]
Now we have:

\[
V_{NSP} = \sum_{l=1}^{D_{ bfd, 3}} l_{ bfd_1}(1.00938712)^{0.9500000} + l_{ bfd_2}(1.00938712)^{0.7222222} + \ldots
\]

\[
+ l_{ bfd_3}(1.00938712)^{0.2914815} + l_{ fd_4} + \sum_{l=1}^{D_{ afd, 16}} \frac{l_{ afd_5}}{(1.0065682)^{0.0555556}} + \ldots
\]

\[
+ \frac{l_{ afd_6}}{(1.0065682)^{1.6666667}} + \frac{l_{ afd_7}}{(1.0065682)^{2.7777778}} + \frac{l_{ afd_8}}{(1.0065682)^{4.2592593}} + \ldots
\]

\[
+ \frac{l_{ afd_9}}{(1.0065682)^{4.4444444}} + \frac{l_{ afd_{10}}}{(1.0065682)^{5.5555556}} + \frac{l_{ afd_{11}}}{(1.0065682)^{6.2222222}} + \ldots
\]

\[
+ \frac{l_{ afd_{12}}}{(1.0065682)^{6.7037037}} + \frac{l_{ afd_{13}}}{(1.0065682)^{7.2962963}} + \frac{l_{ afd_{14}}}{(1.0065682)^{9.0740747}} + \ldots
\]

\[
+ \frac{l_{ afd_{15}}}{(1.0065682)^{10.00000}} + \frac{l_{ afd_{16}}}{(1.0065682)^{11.00000}} + \frac{l_{ afd_{17}}}{(1.0065682)^{11.8518519}} + \ldots
\]

\[
+ \frac{l_{ afd_{18}}}{(1.0065682)^{12.9166667}} + \frac{l_{ afd_{19}}}{(1.0065682)^{13.7107037}} + \frac{l_{ afd_{20}}}{(1.0065682)^{18.5185185}}
\]

Moreover we have

\[
V_{NSP} = \sum_{l=1}^{D_{ afd, 3}} l_{ bfd_1}(1.00891568) + l_{ bfd_2}(1.00677078) + l_{ bfd_3}(1.00272712) + \ldots
\]

\[
+ l_{ fd_4} + \sum_{l=1}^{D_{ afd, 16}} \frac{l_{ afd_{5}}}{(1.00036377)} + \frac{l_{ afd_{6}}}{(1.01097095)} + \frac{l_{ afd_{7}}}{(1.0183517)} + \ldots
\]

\[
+ \frac{l_{ afd_{8}}}{(1.02827659)} + \frac{l_{ afd_{9}}}{(1.02952399)} + \frac{l_{ afd_{10}}}{(1.03704019)} + \frac{l_{ afd_{11}}}{(1.04157623)} + \ldots
\]

\[
+ \frac{l_{ afd_{12}}}{(1.04486459)} + \frac{l_{ afd_{13}}}{(1.04892605)} + \frac{l_{ afd_{14}}}{(1.06120542)} + \frac{l_{ afd_{15}}}{(1.06765775)} + \ldots
\]

\[
+ \frac{l_{ afd_{16}}}{(1.07467034)} + \frac{l_{ afd_{17}}}{(1.08068035)} + \frac{l_{ afd_{18}}}{(1.08824014)} + \frac{l_{ afd_{19}}}{(1.09391191)} + \frac{l_{ afd_{20}}}{(1.12889087)}
\]
Therefore, we obtain the individual coefficients

\[ V_{NSP} = \sum_{i=1}^{3} D_{n3} + \sum_{i=1}^{16} D_{n6} \]

\[ \ldots + 0.99963636 + 0.98914811 + 0.98197901 + 0.97250098 + \ldots \]

\[ \ldots + 0.97132268 + 0.96428278 + 0.96008336 + 0.95706182 + 0.95335605 + \ldots \]

\[ \ldots + 0.94232462 + 0.93662974 + 0.93051791 + 0.925343 + 0.91891482 + \ldots \]

\[ \ldots + 0.91415039 + 0.88582522 \]

Finally, in order to obtain the value of equal payment:

\[ Y = \frac{1,414.00}{19.2214904} \]

\[ Y = 73.5634943 \]

\[ Y = 73.56 \]

**4 Conclusions**

In the illustrative example that we set, the original debt was of $1,420.50. The new scheme of payment is $73.56 * 20 = $1,471.20 and the difference is $50.70, which is the dividend earned by the creditor. As we saw in this paper, a financial strategy that could favor an equitable settlement for all parties involved may be a debt restructuring models using equivalent equation.

The aim of this model was primarily based on three stages. First, we valued the original debt, i.e., considering all the promissory notes that were not paid on the due date and the promissory notes that are pending to be paid. Secondly, we established a new set of payment, i.e., we establish the amount of each payment and more importantly, the dates of maturity. Thus, on this basis, we can identify the coefficient that we need to divide the value of \( V_{OD} \) between \( V_{NSP} \), and thereby we get the amount of each equal payment. If the debtor goes into payment default, this generates the payment of penalty interest on his charge; hence, the creditor would benefit obtaining this profit or dividend.
Similarly, the debtor may have benefit at the moment to use the discount factor to reduce the future value of the debts that are not required. In this way, when we bring to the focal date all of the flows of indexed and discounted promissory notes that have been valued, then we find a fair balance between the apportionment of interest rates and times, which finally benefits both creditor and debtor.

Acknowledgements

The authors are very grateful to the anonymous referees for all suggestions, as well as Universidad Cristobal Colón in Veracruz and Escuela Superior de Economía del Instituto Politécnico Nacional for support. And very special thanks to Felipe de Jesús Pozos Texon, a Ph.D student in the doctoral program at UCC, for helping us to get the Latex format of this paper.

References

Moore, Justin H., 1963. Financial Mathematics. UTEHA. XV,